Engineering Analytic Principles and Predictive Computational Skills for K-12 Students:

Statistics on High School Age-Possible Fluid Mechanics Topics for Engineering and Technology Educators and Curriculum Developers

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Introduction

In the most recent decade, middle and high schools across the United States have tried to incorporate engineering design into traditional technology curriculum, with various degrees of success; however, "the fragmented focus and lack of a clear curriculum framework" had been "detrimental to the potential of the field and have hindered efforts aimed at achieving the stated goals of technological literacy for all students" (Smith and Wicklein, 2007, pp. 2-3). A report issued on September 8, 2009, by the Committee on K-12 Engineering Education established by the National Academy of Engineering and the National Research Council, titled *Engineering in K-12 Education: Understanding the Status and Improving the Prospects* (2009), confirmed the existence of similar problems, such as the "absence of a clear description of which engineering knowledge, skills, and habits of mind are most important, how they relate to and build on one another, and how and when (i.e., at what age) they should be introduced to students" (pp. 7-8; p. 151). K-12 engineering curriculum in the United States remains skeletal so far; its main focus is on generic design process using a "trial-and-error" approach; and the coverage of analytic and predictive knowledge contents is generally in an "ad hoc" fashion and not sequentially structured. In response to the above problems, many scholars have voiced their points of view. Hacker (2011) pointed out that "trial-and-error problem solving takes substantial classroom time, and often does not allow teachers and students to focus on the most important learning goals." Lewis (2007, pp. 846-848) discussed the need to: (a). establish a "codified body of knowledge that can be ordered and articulated across the grades" instead of short term efforts focused on a particular topic or unit, and (b). make engineering education a coherent system with the creation of content standards for the subject area, in line with science and technology education.

High School Age-possible Engineering Topics (Fluid Mechanics)

Research Questions and Practical Conceptual Framework

The above evaluation of the current status of K-12 engineering education in the United States could lead to these questions: (1). "How could we determine what engineering analytic principles and predictive skills from what subject should be taught to students at what Grade in the K-12 curriculum, in a rational and scientific way?" (2). "How could we make sure that what students learned from high school engineering curriculum could be transferred to university programs?" Based on the way engineering curriculum has been historically developed, I have constructed a practical conceptual framework to answer the above two questions. If we read any typical information sheet for university level undergraduate engineering program, we will see that the courses are organized in a sequence based on the fulfillment of pre-requisites in mathematics, physics, chemistry, technology and previous engineering courses; and these pre-requisites are usually listed in course descriptions. Therefore, we could hypothesize that the same principles used historically in the development of curricular structure in university undergraduate engineering programs could apply to the selection of K-12 age-possible engineering analytic principles and predictive skills for any particular Grade, and for any particular subject of engineering. In addition, based on the fact that university undergraduate engineering textbooks, especially those used in foundation courses (such as statics, dynamics, strength of materials, engineering economics, etc.), all contain portions that are based on pre-calculus mathematics and scientific principles which are usually covered in K-12 mathematics and science courses, we could also hypothesize that these pre-calculus portions of engineering topics could possibly be taught at various Grade levels, provided that the pre-requisite pre-calculus mathematics and science principles have been covered in previous Grade levels (or in some cases, taught as special topics); and the coverage of such pre-requisites are usually mandated by the performance standards in mathematics and science established by any particular state. This conceptual framework has been used as a practical tool for the initial determination of 9^{th} grade age-possible statics and fluid mechanics topics. The step-by-step procedure or the “ideal” procedure (Locke, 2009a, pp. 26-27) includes the following (*Figure 1*): (1) selection of data source (selection of popular university undergraduate engineering textbooks and other instructional and learning materials); (2) analysis of data source (careful reading of every paragraph in the body text as well as relevant computational formulas to find and record the pre-requisite mathematics skills and scientific principles needed for each topic; (3) comparison (between the recorded mathematics and science pre-requisites, and my interpretation of the mandates of the Performance Standards for Mathematics and Sciences of the Department of Education of a selected state, in this case, the State of Georgia, to determine the Grade level for the age-possible inclusion of the topics). I selected the State of Georgia’s Standards as a reference for the research because (1) the University of Georgia, my alma mater, gave me the opportunity to study the subject of K-12 engineering education and (2) many professors at the College of Education and the College of Agricultural and Environmental Sciences (Department of Biological and Agricultural Engineering) offered me valuable advice and criticism. Due to the fact that the variations among the K-12 mathematics and science performance standards of the 50 states are not substantial, the outcomes of the research should apply to other states with some reasonable adaptations.

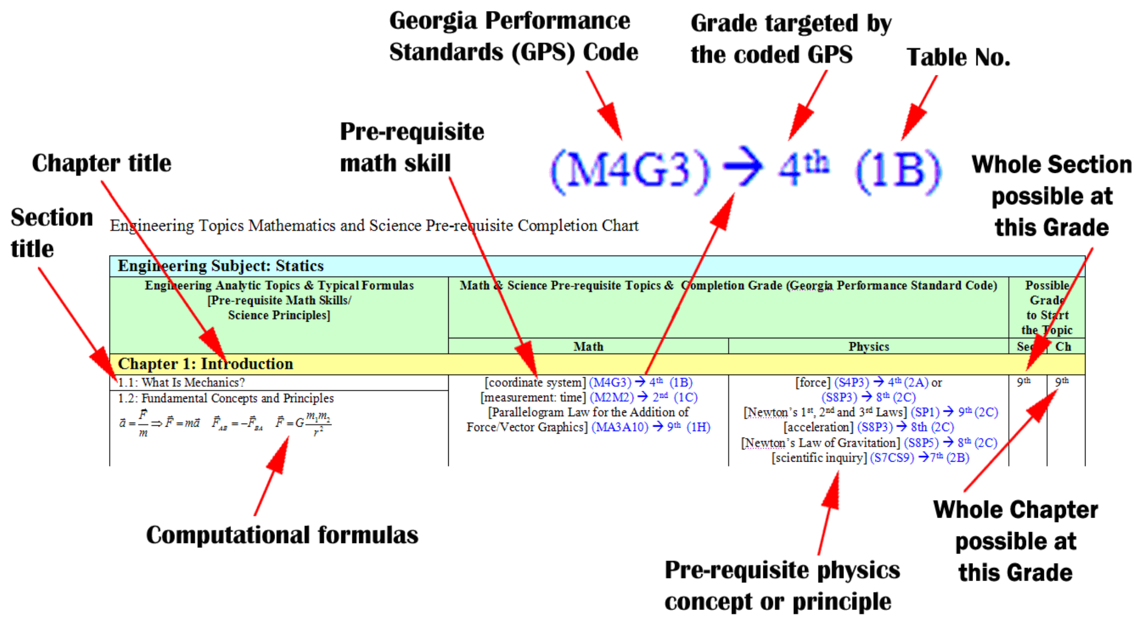

Figure 1. The original research data table used to initially determine high school 9th Grade age-possible statics topics.

After completing all lower-division undergraduate mechanical engineering courses plus two upper-division ones, and conducting a careful and fairly thorough examination of many other college-level engineering textbooks, I have made the conclusion that all engineering textbooks include the following major elements:

(1) **<u>Descriptive and informational:</u>** Paragraphs, data tables, charts, graphs, illustrations and photos that explain natural phenomena, scientific principles, properties of materials, behaviors of structures and systems, in "plain English," without going into the details of analytic and predictive computations using formulas based on mathematics skills.

(2) **<u>Analytic and predictive:</u>** Mathematics-based formulas, including those used in pre-requisite physics and chemistry concepts, principles and analysis, and those used in engineering analysis and design, and step-by-step procedures, including sample problems with solutions, for analyzing problems, predicting outcomes, or designing systems or products; and these mathematics

skills could be at either pre-calculus level, i.e., arithmetic, trigonometry, geometry, algebra, or at calculus level, i.e., integration and differentiation.

In terms of the relative amount of each of the above major elements in the overall composition of the content of the textbooks, all sets of college-level engineering textbooks used in any particular course or subject could be classified into three major categories; i.e., (1) Mixture of Pre-calculus and Calculus, (2) Heavily Pre-calculus, and (3) Heavily Descriptive and Informational. It takes different amounts of time and efforts to examine different sets of textbooks under different categories in order to tentatively determine and select K12 age-possible engineering content knowledge and skills, including descriptive and informational materials, analytic and predictive computational formulas and step-by-step problem solving procedures; and the procedure of this examination include (a) interpretation of the mandates of the Performance Standards for Mathematics and Sciences of the Department of Education of a selected state, in this case, the State of Georgia, to create a coded list of items of mathematics, physics and chemistry concepts and skills, such as *M4G3→4th Grade (1B)* shown in *Figure 1*, from the original online government document, to be used for comparison with the mathematics, physics and chemistry concepts and skills found from the relevant textbooks; (b) paragraph-by-paragraph or page-by-page examination of the selected textbooks for the extraction and documentation of the mathematics, physics and chemistry concepts and skills needed to understand the content and to solve homework problems; and (c) comparison between the interpreted, itemized and coded lists of Performance Standards and the items extracted from the textbooks, to tentatively determine and select sections and chapters in the textbooks that could be K12 age-possible. In the United States, we have a very decentralized management structure for the publication and adaptation of textbooks; and this is especially true for textbooks used in the institutions of higher education where professors usually select textbooks out of their own choices free from government intervention; for any college courses or subject, we can find several excellent and popular textbooks, all of them cover a majority of similar topics plus a small number of different ones; therefore, to be holistic and comprehensive, at least two of the most popular textbooks will be used, one as the "primary source of data" and the rest as "secondary source of data" and "additional sources of data." The nature of composition of the above-mentioned three major categories of textbooks and the average amount of time it takes for their examination are as follows:

1. **Mixture of Pre-calculus and Calculus:** Textbooks under this category include, for the undergraduate mechanical engineering major, those used in the courses of statics, dynamics, strength of materials, electric machines, mechanical design, aerodynamics, fluid mechanics, electrical circuits, heat transfer, thermodynamics, and others. For these textbooks, calculus and pre-calculus skills are used intermittently throughout substantial portions of most of the chapters. These textbooks are usually voluminous and the numbers of pages range from 600 to 900. Therefore, a thorough investigation of all paragraphs, formulas, and even sample problems in the textbooks, and a very detailed record of all pertinent information in tabular forms is needed to determine and to select K-12 age-possible engineering topics for different grade levels. My research projects on the subjects of statics and fluid mechanics have been completed this way. This procedure is very thorough and time-consuming and for one subject, it takes

between 3 to 5 weeks for one textbook (the "primary source of data"), and additional 1 to 2 weeks for another textbook (the "secondary source of data" used to pick up additional K12 age-possible topics); these amounts of time cover careful reading of all chapters, sections, and even appendices and glossaries throughout the entire textbooks, analysis and recording of mathematics, physics and chemistry concepts and skills involved, typing of titles of chapters, sections, formulas, names of pre-requisite items, write-ups of conclusions, as well as a section-by-section review. Typing of titles of chapters, sections, and formulas could take up to one third of the above amounts of time needed for the research. It is the exact or "ideal" procedure advocated in my published Vision Paper.

2. **Heavily Pre-calculus:** Textbooks under this category include those used in the courses of engineering economics, probability and statistics, and others. For these textbooks, the mathematics skills involved in the majority or even the overwhelming majority of chapters and sections are at pre-calculus level; the calculus skills involved in a few sections or chapters are the very beginning ones such as [first integral] and [first derivative]; and the principles and skills related to physics and chemistry are also the very basic ones; therefore, a less time-consuming approach is used to determine and select K12 age-possible engineering topics, by carefully examine each page in the textbooks to record (1) the pre-calculus level mathematics skills as well as physics and chemistry concepts, principles and skills found in all pages; (2) the calculus-level mathematics skills found in some pages, the page numbers where these calculus skills are found, the numbers and names as well as the pages ranges of the sections involving the calculus skills; and (3) result of comparison between the pre-calculus skills as well as physics and chemistry concepts and skills found throughout the textbooks, and the mandates of the Performance Standards for Mathematics and Sciences of the Department of Education of a selected state, in this case, the State of Georgia, to determine the earliest Grade level for the age-possible inclusion of the topics. My research projects on the subjects of engineering economics, probability and statistics, and engineering materials have been completed this way. This procedure is fairly thorough but much less time-consuming because no record of mathematics-based formulas or typing of the names of chapters and sections of the textbooks that involve only pre-calculus mathematics skills is needed, and for one subject, it takes between 5 to 7 days for one textbook (the "primary source of data") and additional 2 to 4 days for another textbook (the "secondary source of data"). These amounts of time cover careful reading of all chapters, sections, and even appendices and glossaries throughout the entire textbooks, analysis and recording of mathematics, physics and chemistry concepts and skills involved as pre-requisites, typing of page numbers and titles of chapters and sections involving calculus stills as well as numbers of the individual pages involved, write-ups of conclusions, as well as a section-by-section review. It is a convenient and "ad hoc" revision of the "ideal" procedure advocated in my published Vision Paper.

3. **Heavily Descriptive and Informational:** Textbooks under this category include those used in the courses of introduction to science, engineering and technology, ethics and professionalism in engineering, and others. These textbooks involve little or no

mathematics skills; their primary goal is to expose students to broad knowledge about engineering, science, technology, as well as their relationship with “other stuff” such as society, ecology, legal system, philosophy, and others. Similar method as the one used for the “Heavily Pre-calculus” textbooks is used here but the amounts of time spent is substantially reduced because, for the “Heavily Descriptive and Informational” textbook, mathematics, physics and chemistry pre-requisites are rarely involved. For one subject, it takes between 1 to 3 days for one textbook (the “primary source of data”) and additional 1 to 2 days for another textbook (the “secondary source of data”). These amounts of time cover careful reading of all chapters, sections, and even appendices and glossaries throughout the entire textbooks, analysis and recording of a few mathematics, physics and chemistry concepts and skills involved as pre-requisites, write-ups of conclusions, as well as a section-by-section review. It is a convenient, “ad hoc” and more drastic revision of the “ideal” procedure advocated in my published Vision Paper.

Sources of Data

Table 1 lists (1) the college-level Textbook 1 (the “primary source of data”) used for the extraction of analytic and predictive principles and computational formulas related to the subject of fluid mechanics, and (2) the instructor’s or student’s solution manuals used to double-check for the mathematics computational skills needed for the study of various topics of fluid mechanics contained in the selected.

Table 1. Data Source (Fluid Mechanics Textbook)

	Main Textbook	**Reference Book**	**Student Solution Manual**
Title	Fundamentals of Fluid Mechanics Mechanics, 5th Edition	A Brief Introduction to Fluid Mechanics, 4th Edition	A Brief Introduction to Fluid Mechanics, Student Solutions Manual, 4th Edition
Authors	Bruce M. Munson, Donald F. Young, Theodore H. Okiishi	Donald F. Young, Bruce R. Munson, Theodore H. Okiishi, Wade W. Huebsch	Donald F. Young, Bruce R. Munson, Theodore H. Okiishi, Wade W. Huebsch
Publisher	John Wiley & Sons, Inc.	John Wiley & Sons, Inc.	John Wiley & Sons, Inc.
Year	2006	2007	2007
ISBN	0-471-67582-2	978-0470039625	978-0470099285

Notes for Chapter 6 and Chapter 7

Chapter 6 (Differential Analysis of Fluid Mechanics Flow) appears to be, for all practical purposes, too deep in calculus-based mathematics for even 12^{th} Grade students in Advanced Placement Calculus course to master.

Chapter 7 (Similitude, Dimensional Analysis, and Modeling) involves a lot of "abstract thinking" and appears to be most likely beyond the cognitive developmental maturity level of high school students.

Therefore, engineering analytic principles and skills from these two Chapters are NOT analyzed for the eventual inclusion into a potentially viable K-12 engineering curriculum. However, some generic knowledge content covered in these two Chapters could still be lightly explored by 9^{th} or above Grade students; thus, their relative importance could still be rated at generic knowledge level. In addition, some appropriate skills in 7.1 (Dimensional Analysis) could be considered for high schools.

Original Research Data

Table 2 (Fluid Mechanics Topic List) constitute the original research data; the way data is recorded and analyzed is shown in *Figure 1*. The leftmost column in the Table 1 (Fluid Mechanics Topic List - Engineering Topics Mathematics and Science Pre-requisite Completion Chart for the Subject of Fluid Mechanics) contains

1. The titles of each section under a particular chapter in the selected textbook, which in general represent particular sets of fluid mechanics related engineering analytic and predictive principles, in a qualitative and explanatory way;

2. Computational formulas, which symbolically represent the above engineering analytic and predictive principles, in a quantitative and mathematical way.

The middle column (divided into two sub-columns, i.e., Math, and Physics/Chemistry, is reserved for recording mathematics and science (physics and chemistry) pre-requisites for understanding the knowledge content and using the formulas contained in the particular chapters or sections, as well as the grade levels these pre-requisites are expected to be fulfilled according to Georgia Performance Standards.

The right column records the prediction on the possible grade a particular chapter or section could be taught to K12 students.

Findings from the Research Project

Table 3A (Pre-Calculus Based Fluid Mechanics Topics That Possibly Could Be Taught at 9^{th} Grade), Table 3B (Pre-Requisite Mathematics and Science Topics to Be Reviewed Before Teaching the Pre-Calculus Portion of Fluid Mechanics Topics to 9^{th} Grade Students), Table 4A (Calculus Based Fluid Mechanics Topics for Post-Secondary Engineering Education), and Table 4B (Pre-Requisite Math and Science Topics to Be Reviewed Before Teaching the Calculus Portion of Fluid Mechanics Topics) constitute the outcomes of the research, which is presented here to the public as "public domain" knowledge, to be used as sources of reference by K12 engineering and technology education practitioners and curriculum developers, nationally and internationally, including myself, of course!

Table 2. Fluid Mechanics Topic List
Engineering Topics Mathematics and Science Pre-requisite Completion Chart for the Subject of Fluid Mechanics

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code)** **[Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 1 - Introduction				
1.1 Some Characteristics of Fluid	N/A	[pressure] (SC5) → 9th (4B) → To be taught [velocity] (S8P3)→ 8th (3A) [force] (S4P3) → 4th (3A) or (S8P3) → 8th (3C) [molecule] (S8P1)→ 8th (4A)	9th	9th + PS
1.2 Dimensions, Dimensional Homogeneity, and Units $p \equiv \frac{\vec{F}_n}{A_s} \rightarrow \vec{F}_n = pA_s \quad \tau = \frac{P}{A_s} \quad \tau \propto \delta\beta$	[unit conversion] (M6M1) → 6th (2C) [four operations] (M1N3) → 1st (2A) [square root] (M8N1) → 8th (2A)	N/A	9th	
1.3 Analysis of Fluid Mechanics Behavior N/A	N/A	[Newton's 1st, 2nd and 3rd Laws] (SP1) → 9th (3C) [mass] (S8P3) → 8th (3A)	9th	
1.4 Measures of Fluid Mechanics Mass and Weight	[four operations] (M1N3) → 1st (2A)	[mass] (S8P3) → 8th (3A) [gravity] (S6E1) → 6th (3A)	9th	
1.4.1 Density $\rho = \frac{m}{V} \quad v = \frac{V}{m} = \frac{1}{\rho}$	[four operations] (M1N3) → 1st (2A) [volume] (M5M4) → 5th (1B) (M6M3) → 6th (2B) (MA1G5) → 9th (2F)	[density] (S6E5) → 6th (4A) [mass] (S8P3) → 8th (3A)	9th	
1.4 2 Specific Weight $\gamma \equiv \frac{W}{V} = \frac{mg}{V} = \rho g$	[four operations] (M1N3) → 1st (2A)	[force] (S4P3) → 4th (3A) or (S8P3) → 8th (3C) [gravity] (S6E1) → 6th (3A) [density] (S6E5) → 6th (4A)	9th	
1.4.3 Specific Gravity $SG = \frac{\rho}{\rho_{H_2O} @ 4^\circ C}$	[four operations] (M1N3) → 1st (2A)	[pressure] (SC5) → 9th (4B) → To be taught	9th	
1.5 Ideal Gas Law $p = \rho RT$	[four operations] (M1N3) → 1st (2A)	[temperature] (SP3) → 9th (3B) [**absolute temperature**] (SP3) → 9th (3B) → To be taught [density] (S6E5) → 6th (4A)	9th	

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code)** **[Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 1 – Introduction (Continued)				
1.6 Viscosity $u=\frac{Uy}{b}\quad \tan\delta\beta \approx \delta\beta = \frac{\delta a}{b}$ $\delta a = U\delta t\quad \delta\beta=\frac{U\delta t}{b}\quad \dot{\gamma}=\lim_{\delta t\to 0}\frac{\delta\beta}{\delta t}\quad \dot{\gamma}=\frac{U}{b}=\frac{du}{dy}$ $\tau \propto \dot{\gamma}\quad \tau \propto \frac{du}{dy}\quad \tau=\mu\frac{du}{dy}$ $\mu=\frac{CT^{3/2}}{T+S}\quad \mu=De^{B/T}\quad \nu=\frac{\mu}{\rho}$	[four operations] (M1N3) → 1st (2A) [derivative] → 12th (To be taught) [trigonometric functions] (MA2G2) → 10th (2F)	[density] (S6E5) → 6th (4A) [**absolute temperature**] (SP3) → 9th (3B) → To be taught	PS	9th + PS
1.7 Compressibility of Fluids	N/A	N/A	9th	
1.7.1 Bulk Modulus $E_v = \frac{dp}{d\forall/\forall}\quad E_v=\frac{dp}{d\rho/\rho}$	[four operations] (M1N3) → 1st (2A) [derivative] → 12th (To be taught as a special skill)	[pressure] (SC5) → 9th (4B) → To be taught [density] (S6E5) → 6th (4A)	9th	
1.7.2 Compression and Expansion of Gases $\frac{p}{\rho}=\text{Constant}\quad \frac{p}{\rho^k}=\text{Constant}\quad E_v=p\quad E_v=kp$	[four operations] (M1N3) → 1st (2A) [exponent] (M6A3) → 6th (2A)	[pressure] (SC5) → 9th (4B) → To be taught [density] (S6E5) → 6th (4A)	9th	
1.7.3 Speed of Sound $c=\sqrt{\frac{dp}{d\rho}}=\sqrt{\frac{E_v}{\rho}} \leftarrow \begin{cases} E_v=\frac{dp}{d\rho/\rho}=\frac{dp}{d\rho}\rho \\ \frac{E_v}{\rho}=\frac{dp}{d\rho} \end{cases}$ $\left.\begin{matrix} c=\sqrt{\frac{kp}{\rho}} \\ p=\rho RT \end{matrix}\right\} \to \quad c=\sqrt{kRT}$	[four operations] (M1N3) → 1st (2A) [square root] (M8N1) → 8th (2A) [derivative] → 12th (To be taught as a special skill)	[speed of sound] (SPS9) → 9th (3B) → To be taught [velocity] (S8P3)→ 8th (3A) [**absolute temperature**] (SP3) → 9th (3B) → To be taught	9th	

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 1 – Introduction (Continued)				
1.8 Vapor Pressure	N/A	**[intermolecular cohesive force]** → To be taught [momentum] (SP3) → 9th (3B) [pressure] (SC5) → 9th (4B) → To be taught	9th	9th + PS
1.9 Surface Tension $2\pi R\sigma = \Delta p \pi R^2 \quad \Delta p = p_i - p_e = \dfrac{2\sigma}{R}$ $\gamma \pi R^2 h = 2\pi R\sigma \cos\theta \quad \rightarrow \quad h = \dfrac{2\sigma\cos\theta}{\gamma R}$	[areas of geometric shapes: circle, triangle] (M5M1) → 5th (2B) (M5M1) → 5th (2B) [unit conversion] (M6M1) → 6th (2C) [height] (MKM1) → K (2B) [trigonometric functions] (MA2G2) → 10th (2F)	[force] (S4P3) → 4th (3A) or (S8P3) → 8th (3C) [mass] (S8P3) → 8th (3A) [pressure] (SC5) → 9th (4B) → To be taught [weight] (MKM1) → K (2C) [gravity] (S6E1) → 6th (3A)	9th	
1.10 A Brief Look Back in History	N/A	N/A	9th	
1.11 Chapter Summary and Study Guide	N/A	N/A	9th	
Chapter 2 Fluid Statics (Continued)				
2.1 Pressure at a Point $\vec{F} = m\vec{a}$ $\sum F_y = p_y \delta x \delta z - p_s \delta x \delta s \sin\theta = \rho \dfrac{\delta x \delta y \delta z}{2} a_y$ $\sum F_z = p_z \delta x \delta y - p_s \delta x \delta s \cos\theta - \gamma \dfrac{\delta x \delta y \delta z}{2} = \rho \dfrac{\delta x \delta y \delta z}{2} a_z$ $\uparrow \quad \dfrac{\delta x \delta y \delta z}{2} = V \quad m = \rho V \quad \leftarrow \quad \vec{F}_z = m\vec{a}_z$	[four operations] (M1N3) → 1st (2A) [sigma notation] (M6N1) → 6th (1A) or (MA1A3) → 9th (2E) [coordinate system] (M4G3) → 4th (2B) [limit] → Post-Secondary [volume] (M5M4) → 5th (1B) (M6M3) → 6th (2B) (MA1G5) → 9th (2F)	[force] (S4P3) → 4th (3A) or (S8P3) → 8th (3C) [Newton's 1st, 2nd and 3rd Laws] (SP1) → 9th (3C)	PS	PS

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 2 Fluid Statics (Continued)				
2.1 Pressure at a Point (Continued) $\vec{F} = m\vec{a}$ $\sum F_y = p_y \delta x \delta z - p_s \delta x \delta s \sin\theta = \rho \frac{\delta x \delta y \delta z}{2} a_y$ $\sum F_z = p_z \delta x \delta y - p_s \delta x \delta s \cos\theta - \gamma \frac{\delta x \delta y \delta z}{2} = \rho \frac{\delta x \delta y \delta z}{2} a_z$ $\uparrow \frac{\delta x \delta y \delta z}{2} = V \quad m = \rho V \quad \leftarrow \quad \vec{F}_z = m\vec{a}_z$	[four operations] (M1N3) → 1st (2A) [sigma notation] (M6N1) → 6th (1A) or (MA1A3) → 9th (2E) [coordinate system] (M4G3) → 4th (2B) [limit] → Post-Secondary [volume] (M5M4) → 5th (1B) (M6M3) → 6th (2B) (MA1G5) → 9th (2F)	[force] (S4P3) → 4th (3A) or (S8P3) → 8th (3C) [Newton's 1st, 2nd and 3rd Laws] (SP1) → 9th (3C)	PS	PS
2.2 Basic Equation for Pressure Field $\delta F_y = \left(p - \frac{\partial p}{\partial y}\frac{\delta y}{2}\right)\delta x \delta z - \left(p + \frac{\partial p}{\partial y}\frac{\delta y}{2}\right)\delta x \delta z \quad \rightarrow$ $\left.\begin{aligned} \delta F_x &= -\frac{\partial p}{\partial x}\delta x \delta y \delta z \\ \delta F_y &= -\frac{\partial p}{\partial y}\delta x \delta y \delta z \\ \delta F_z &= -\frac{\partial p}{\partial z}\delta x \delta y \delta z \end{aligned}\right\} \quad \leftarrow \quad \delta y \delta x \delta z = V$	[four operations] (M1N3) → 1st (2A) [partial derivative] → Post-Secondary [gradient "del"] → Post-Secondary [volume] (M5M4) → 5th (1B) (M6M3) → 6th (2B) (MA1G5) → 9th (2F) [coordinate system] (M4G3) → 4th (2B)	[pressure] (SC5) → 9th (4B) → To be taught [acceleration] (S8P3) → 8th (3C)	PS	

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 2 Fluid Statics (Continued)				
2.2 Basic Equation for Pressure Field (Continued) $\delta \vec{F}_s = \delta F_x \hat{i} + \delta F_y \hat{j} + \delta F_z \hat{k} = -\left(\frac{\partial p}{\partial x}\hat{i} + \frac{\partial p}{\partial y}\hat{j} + \frac{\partial p}{\partial z}\hat{k}\right)\delta x \delta y \delta z$ $\frac{\partial p}{\partial x}\hat{i}+\frac{\partial p}{\partial y}\hat{j}+\frac{\partial p}{\partial z}\hat{k} = \nabla p \quad \nabla(\ ) = \frac{(\ )}{\partial x}\hat{i}+\frac{(\ )}{\partial y}\hat{j}+\frac{(\ )}{\partial z}\hat{k}$ $\frac{\delta \vec{F}_s}{\delta x\delta y\delta z} = -\nabla p \quad -\delta W\hat{k} = -\gamma \delta x \delta y \delta z \hat{k} \quad \begin{cases}\sum \delta\vec{F} = \delta m\vec{a} \\ \delta m = p\delta x\delta y\delta z\end{cases}$ $\sum \delta\vec{F} = \delta\vec{F}_s - \delta W\hat{k} = \delta m\vec{a} \rightarrow$ $-\nabla p\delta x\delta y\delta z\hat{k} - \gamma\delta x\delta y\delta z\hat{k}\hat{k} = \rho\delta x\delta y\delta z\hat{k}\vec{a} \rightarrow$ $-\nabla p - \gamma\hat{k} = \rho\vec{a}$	[four operations] (M1N3) → 1st (2A) [partial derivative] → Post-Secondary [gradient "del"] → Post-Secondary [volume] (M5M4) → 5th (1B) (M6M3) → 6th (2B) (MA1G5) → 9th (2F) [coordinate system] (M4G3) → 4th (2B)	[pressure] (SC5) → 9th (4B) → To be taught [acceleration] (S8P3) → 8th (3C)	PS	PS
2.3 Pressure Variation in a Fluid Mechanics at Rest $\vec{a} = 0 \quad \rightarrow \quad -\nabla p - \gamma\hat{k} = 0$ $\frac{\partial p}{\partial x} = 0 \quad \frac{\partial p}{\partial y} = 0 \quad \frac{\partial p}{\partial z} = -\gamma \quad \rightarrow \quad \frac{dp}{dz} = -\gamma$	[four operations] (M1N3) → 1st (2A) [derivative] → 12th (To be taught) [partial derivative] → Post-Secondary [gradient] → Post-Secondary	[pressure] (SC5) → 9th (4B) → To be taught	PS	

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)		Possible Grade to Start the Topic	
	Math	Physics/Chemistry	Sec	Ch
Chapter 2 Fluid Statics (Continued)				
2.3.1 Incompressible Fluid $\int_{p_1}^{p_2} dp = -\gamma \int_{z_1}^{z_2} dz \rightarrow \begin{cases} p_2 - p_1 = -\gamma(z_2 - z_1) \\ p_1 - p_2 = \gamma(z_2 - z_1) \end{cases}$ $\rightarrow p_1 - p_2 = \gamma h \rightarrow \begin{cases} p_1 = \gamma h + p_2 \\ h = \dfrac{p_1 - p_2}{\gamma} \end{cases}$ $p = \gamma h + p_0 = \rho g h + p_0$	[four operations] (M1N3) → 1st (2A) [integration] → 12th (To be taught) Note: The main Formula $p = \gamma h + p_0 = \rho g h + p_0$ does not need calculus.	[pressure] (SC5) → 9th (4B) → To be taught	9th	9th
2.3.2 Compressible Fluid $\left. \begin{array}{l} p = \rho RT \\ \rho = \dfrac{p}{RT} \\ \dfrac{dp}{dz} = -\gamma = -\rho g \end{array} \right\} \dfrac{dp}{dz} = -\dfrac{gp}{RT} \rightarrow$ $(dz)\dfrac{dp}{dz(p)} = -\dfrac{gp}{RT(p)}(dz) \quad \dfrac{dp}{p} = -\dfrac{g}{RT} dz \rightarrow$ $\int_{p_1}^{p_2} \dfrac{dp}{p} = \int_{z_1}^{z_2} -\dfrac{g}{RT} dz = -\dfrac{g}{R} \int_{z_1}^{z_2} \dfrac{dz}{T} \rightarrow$ $\int_{p_1}^{p_2} \dfrac{dp}{p} = \ln \dfrac{p_2}{p_1} = -\dfrac{g}{R} \int_{z_1}^{z_2} \dfrac{dz}{T}$ $p_2 = p_1 \exp\left[-\dfrac{g(z_2 - z_1)}{RT_0} \right]$	[four operations] (M1N3) → 1st (2A) [exponent] (M6A3) → 6th (2A) [integration] → 12th (To be taught as a special skill) [derivative] → 12th (To be taught) Note: The main formula $p_2 = p_1 \exp\left[-\dfrac{g(z_2 - z_1)}{RT_0} \right]$ does not need calculus	[pressure] (SC5) → 9th (4B) → To be taught **[absolute temperature]** (SP3) → 9th (3B) → To be taught **[gas/liquid]** (SPS5) → 9th (3B) → To be taught	9th	

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 2 Fluid Statics (Continued)				
2.4 Standard Atmosphere $T = T_a - \beta z \quad p = p_a\left(1 - \frac{\beta z}{T_a}\right)^{g/R\beta}$	[four operations] (M1N3) → 1st (2A) [exponent] (M6A3) → 6th (2A)	[temperature] (SP3) → 9th (3B) [pressure] (SC5) → 9th (4B) → To be taught [density] (S6E5) → 6th (4A) [weight] (MKM1) → K (2C)	9th	9th + PS
2.5 Measurement of Pressure $p_{abs} = p_{gage} + p_{atm} \quad p_{atm} = \gamma h + p_{vapor}$	[four operations] (M1N3) → 1st (2A)	[pressure] (SC5) → 9th (4B) → To be taught	9th	
2.6 Monometry	[four operations] (M1N3) → 1st (2A) [cylinder] (M1G1) (M1G2) → 1st (2B)	[pressure] (SC5) → 9th (4B) → To be taught	9th	
2.6.1 Piezometer Tube $p = \gamma h + p_0 \quad p_A = \gamma_1 h_1$	[four operations] (M1N3) → 1st (2A) [height] (MKM1) → K (2B)	[pressure] (SC5) → 9th (4B) → To be taught	9th	
2.6.2 U-Tube Manometer $p = \gamma h + p_0 \quad p_A + \gamma_1 h_1 - \gamma_2 h_2 = 0 \quad \rightarrow$ $p_A = \gamma_2 h_2 - \gamma_1 h_1 \quad p_A = \gamma_2 h_2$ $p_A + \gamma_1 h_1 - \gamma_2 h_2 - \gamma_3 h_3 = p_B \quad \rightarrow$ $p_A - p_B = \gamma_2 h_2 + \gamma_3 h_3 - + \gamma_1 h_1$	[four operations] (M1N3) → 1st (2A)	[pressure] (SC5) → 9th (4B) → To be taught	9th	
2.6.3 Inclined-Tube Manometer $p_A + \gamma_1 h_1 - \gamma_2 \ell_2 \sin\theta - \gamma_3 h_3 = p_B$ $p_A - p_B = \gamma_2 \ell_2 \sin\theta + \gamma_3 h_3 - \gamma_1 h_1$ $p_A - p_B = \gamma_2 \ell_2 \sin\theta \quad \rightarrow \quad \ell_2 = \frac{p_A - p_B}{\gamma_2 \sin\theta}$	[four operations] (M1N3) → 1st (2A) [trigonometric functions] (MA2G2) → 10th (2F)	[pressure] (SC5) → 9th (4B) → To be taught	9th	
2.7 Mechanical and Electronic Pressure Measuring Devices	N/A	N/A		

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code)** **[Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 2 Fluid Statics (Continued)				
2.8 Hydrostatic Force on a Plane Surface $F_R = \int_A \gamma h\, dA = \int_A \gamma y \sin\theta\, dA \quad F_R = \gamma \sin\theta \int_A y\, dA$ $\int_A y\, dA = y_c A \quad F_R = \gamma A y_c \sin\theta$ $F_R = \lambda h_c A \quad F_R y_R = \int_A y\, dF = \int_A \gamma \sin\theta\, y^2 dA$ $y_R = \frac{\int_A y^2 dA}{y_c A} = \frac{I_x}{y_c A} = \frac{I_{xc} + A y_c^2}{y_c A}$ $y_R = \frac{I_{xc}}{y_c A} + \frac{A y_c^2}{y_c A} = \frac{I_{xc}}{y_c A} + y_c \quad \leftarrow \quad I_x = I_{xc} + A y_c^2$ $F_R x_R = \int_A \gamma \sin\theta\, xy\, dA \quad x_R = \frac{\int_A xy\, dA}{y_c A} = \frac{I_{xy}}{y_c A}$ $x_R = \frac{I_{xyc}}{y_c A} + x_c \quad I_{xy} = I_{xyc} + A x_c y_c$	[surface] (M6M4) → 6th (2B) [four operations] (M1N3) → 1st (2A) [exponent] (M6A3) → 6th (2A) [trigonometric functions] (MA2G2) → 10th (2F) [integration] → 12th (To be taught as a special skill)	[force] (S4P3) → 4th (3A) or (S8P3) → 8th (3C) [weight] (MKM1) → K (2C) [pressure] (SC5) → 9th (4B) → To be taught [1st moment of the area] → To be taught [2nd moment of the area] → To be taught	PS	9th + PS
2.9 Pressure Prism $F_R = p_{av} A = \gamma\left(\frac{h}{2}\right)A \quad F_R = volume = \frac{1}{2}(\gamma h)(bh) = \gamma\left(\frac{h}{2}\right)A$ $F_R = F_1 + F_2 \quad F_R y_A = F_1 y_1 + F_2 y_2$	[four operations] (M1N3) → 1st (2A) [prism] (M6G2) → 6th (2B)	[pressure] (SC5) → 9th (4B) → To be taught [force] (S4P3) → 4th (3A) or (S8P3) → 8th (3C)	9th	
2.10 Hydrostatic Force on a Curves Surface $F_H = F_2 \quad F_v = F_1 + \vec{W}$ $F_R = \sqrt{(F_H)^2 + (F_v)^2}$	[four operations] (M1N3) → 1st (2A) [Pythagorean Theorem] (M8G2) → 8th (2B)	[force] (S4P3) → 4th (3A) or (S8P3) → 8th (3C)	9th	

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 2 Fluid Statics (Continued)				
2.11 Buoyancy, Flotation, and Stability N/A **2.11.1 Archimedes' Principle** $F_B = F_2 - F_1 - \vec{W} \quad F_2 - F_1 = \gamma(h_2 - h_1)A$ $F_B = \gamma(h_2 - h_1)A - \gamma[(h_2 - h_1)A - \forall]$ $F_B = \gamma\forall \quad F_B y_c = F_2 y_1 - F_1 y_1 - \vec{W} y_2$ $\forall y_c = \forall_T y_1 - (\forall_T - \forall) y_2$ **2.11.2 Stability** N/A	[four operations] (M1N3) → 1st (2A) [volume] (M5M4) → 5th (2B) (M5M4) → 5th (1B) (M6M3) → 6th (2B) (MA1G5) → 9th (2F)	[force] (S4P3) → 4th (3A) or (S8P3) → 8th (3C) [weight] (M4M1) → 4th (2C)	9th	9th + PS
2.12 Pressure Variation in a Fluid Mechanics with Rigid-Body Motion $-\nabla p - \gamma\hat{k} = \rho\vec{a} \quad \rightarrow \quad \begin{cases} -\dfrac{\partial p}{\partial x} = \rho a_x \\ -\dfrac{\partial p}{\partial y} = \rho a_y \\ -\dfrac{\partial p}{\partial z} = \gamma + \rho a_z \end{cases}$	[four operations] (M1N3) → 1st (2A) [coordinate system] (M4G3) → 4th (2B) [partial derivative] → Post-secondary [gradient] → Post-secondary	[pressure] (SC5) → 9th (4B) → To be taught [density] (S6E5) → 6th (4A)	PS	
2.12.1 Linear Motion $\dfrac{\partial p}{\partial y} = -\rho a_y \quad \dfrac{\partial p}{\partial z} = -\rho(g + a_z) \quad dp = \dfrac{\partial p}{\partial y}dy + \dfrac{\partial p}{\partial z}dz$ $dp = -\rho a_y dy - \rho(g + a_z)dz$ $\dfrac{dz}{dy} = -\dfrac{a_y}{g + a_z} \quad \dfrac{dp}{dz} = -\rho(g + a_z)$	[four operations] (M1N3) → 1st (2A) [derivative] → 12th (To be taught) [partial derivative] → Post-secondary	[pressure] (SC5) → 9th (4B) → To be taught [density] (S6E5) → 6th (4A)	PS	

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code)** **[Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 2 Fluid Statics (Continued)				
2.12.2 Rigid-Body Rotation $\nabla p = \frac{\partial p}{\partial r}\hat{e}_r + \frac{1}{r}\frac{\partial p}{\partial \theta}\hat{e}_\theta + \frac{\partial p}{\partial z}\hat{e}_z \quad \vec{a}_r = -r\omega^2\hat{e}_r \quad \vec{a}_\theta = 0 \quad \vec{a}_z = 0$ $\frac{\partial p}{\partial r} = \rho r\omega^2 \quad \frac{\partial p}{\partial \theta} = 0 \quad \frac{\partial p}{\partial z} = -\gamma$ $dp = \frac{\partial p}{\partial r}dr + \frac{\partial p}{\partial z}dz \quad dp = \rho r\omega^2 dr - \gamma\ dz$ $dp = 0 \quad \rightarrow \quad 0 = \rho r\omega^2 dr - \gamma\ dz \quad \rightarrow$ $0 = \rho r\omega^2 dr - \rho g\ dz \quad \rightarrow \quad 0 = r\omega^2 dr - g\ dz \quad \rightarrow$ $g\ dz = r\omega^2 dr \quad \rightarrow \quad \frac{dz}{dr} = \frac{r\omega^2}{g} \quad \rightarrow$ $\int \frac{dz}{dr} = \int \frac{r\omega^2}{g} \quad \rightarrow \quad \int \frac{dz}{dr}(dr) = \int \frac{r\omega^2}{g}(dr) \quad \rightarrow$ $\int dz = \int \frac{r\omega^2}{g} dr \quad \rightarrow \quad z = \frac{r^2}{2}\frac{\omega^2}{g} \quad \rightarrow$ $z = \frac{\omega^2 r^2}{2g} + \text{constant} \quad \int \text{dp} = \rho\omega^2 \int r\ dr - \gamma \int dz$ $p = \frac{\rho\omega^2 r^2}{2} - \gamma z + \text{constant}$	[four operations] (M1N3) → 1st (2A) [derivative] → 12th (To be taught) [partial derivatives] → Post-secondary [gradient] → Post-secondary [integration] → 12th (To be taught)	[pressure] (SC5) → 9th (4B) → To be taught [gravity] (S6E1) → 6th (3A) [force] (S4P3) → 4th (3A) or (S8P3) → 8th (3C)	PS	9th + PS
2.13 Chapter Summary and Study Guide	N/A	N/A	**9th**	
Chapter 3 Elementary Fluid Dynamics – The Bernoulli Equation				
3.1 Newton's Second Law $\vec{F} = m\vec{a} \quad \sum(\vec{F}_P + \vec{F}_g) = m\vec{a}$ $a_s = V\frac{\partial V}{\partial s} \quad a_n = V\frac{V^2}{\Re} \quad \leftarrow \quad V = \lvert\vec{V}\rvert$	[four operations] (M1N3) → 1st (2A) [partial derivative] → Post-secondary [volume] (M5M4) → 5th (1B) (M6M3) → 6th (2B) (MA1G5) → 9th (2F) Note: The main formula $\vec{F} = m\vec{a}$ does not need calculus	[Newton's 1st, 2nd and 3rd Laws] (SP1) → 9th (3C) → To be taught [force] (S4P3) → 4th (3A) or (S8P3) → 8th (3C) [speed] (S2P3) → 2nd (3A)	9th	9th + PS

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code)** **[Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 3 Elementary Fluid Dynamics – The Bernoulli Equation (Continued)				
3.2 F = ma along a Streamline $\sum \delta F_s = \delta m a_s = \delta m V \frac{\partial V}{\partial s} = \rho \delta \mathcal{V} V \frac{\partial V}{\partial s}$ $\left. \begin{array}{l} \delta \vec{W} = \gamma \delta \mathcal{V} \\ \gamma = \rho g \end{array} \right\} \rightarrow \delta \vec{W}_s = -\delta \vec{W} \sin\theta = -\gamma \delta X \mathcal{V} \sin\theta$ $\delta p_s \approx \frac{\partial p}{\partial s} \frac{\delta s}{2}$ $\delta F_{ps} = (p - \delta p_s)\delta n \delta y - (p + \delta p_s)\delta n \delta y = -2\delta p_s \delta n \delta y$ $= -\frac{\partial p}{\partial s} \delta s \delta n \delta y = -\frac{\partial p}{\partial s} \delta \mathcal{V}$ $\sum \delta F_s = \delta \vec{W}_s + \delta F_{ps} = \left(-\gamma \sin\theta - \frac{\partial p}{\partial s} \right) \delta \mathcal{V}$ $-\gamma \sin\theta - \frac{\partial p}{\partial s} = \rho V \frac{\partial V}{\partial s} = \rho a_s$ $-\gamma \frac{dz}{ds} - \frac{dp}{ds} = \frac{1}{2} \rho \frac{d(V^2)}{ds} \rightarrow dp + \frac{1}{2} \rho d(V^2) + \gamma \, dz = 0$ $\int \frac{dp}{\rho} + \frac{1}{2} V^2 + gz = C$ (along a streamline) $p + \frac{1}{2} \rho V^2 + \gamma z =$ constant along a streamline (Bernoulli Equation)	[four operations] (M1N3) → 1st (2A) [trigonometric functions] (MA2G2) → 10th (2F) [partial derivative] → Post-secondary [sigma notation] (M6N1) → 6th (1A) or (MA1A3) → 9th (2E) Note: The main formulas $\vec{F} = m\vec{a}$ and $p + \frac{1}{2} \rho V^2 + \gamma z =$ constant along a streamline (Bernoulli Equation) do not need calculus	[force] (S4P3) → 4th (3A) or (S8P3) → 8th (3C) [gravity] (S6E1) → 6th (3A) [mass] (S8P3) → 8th (3A) [acceleration] (S8P3) → 8th (3C)	9th	9th + PS

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code)** **[Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 3 Elementary Fluid Dynamics – The Bernoulli Equation (Continued)				
3.2 F = ma along a Streamline (Continued) Alternatively $p_N + \frac{1}{2}\rho_N \cdot {v_N}^2 + \rho_N \cdot g \cdot z_N = C_{streamline\,N}$ Pressure (along a streamline) + Kinetic Energy + Potential Energy = Constant $\begin{cases} p = \frac{F}{A} = \frac{F \cdot r}{A \cdot r} = \frac{W}{V} \text{ (Work per unit volume)} \\ \frac{1}{2}\rho \cdot v^2 = \frac{1}{2}\frac{m}{\cancel{V}} \cdot v^2 = \frac{\frac{1}{2} m \cdot v^2}{\cancel{V}} = \frac{KE}{\cancel{V}} \text{ (Kinetic energy per unit volume)} \\ \rho \cdot g \cdot z = \frac{m}{\cancel{V}} \cdot g \cdot z = \frac{m \cdot g \cdot z}{\cancel{V}} = \frac{PE}{\cancel{V}} \\ \text{(Potential energy per unit volume. } z\hat{k} \uparrow \equiv h;\ g \equiv -z\hat{k} \downarrow) \end{cases}$ Law of conservation of mass + Law of conservation of energy Bernouilli's Equation $p_N + \frac{1}{2}\rho_N \cdot {v_N}^2 + \rho_N \cdot g \cdot z_N = C_{streamline\,N}$ $(v_{in} \cdot \hat{n}_{Ain}) \cdot A_{in} = A_{2\ out} \cdot v_{2\ out} + A_{1\ out} \cdot v_{1\ out}$	[four operations] (M1N3) → 1st (2A) [trigonometric functions] (MA2G2) → 10th (2F) [dot product] → To be taught as a special math topic [partial derivative] → Post-secondary [sigma notation] (M6N1) → 6th (1A) or (MA1A3) → 9th (2E)	[force] (S4P3) → 4th (3A) or (S8P3) → 8th (3C) [gravity] (S6E1) → 6th (3A) [mass] (S8P3) → 8th (3A) [acceleration] (S8P3) → 8th (3C)	PS	9th + PS

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 3 Elementary Fluid Dynamics – The Bernoulli Equation (Continued)				
3.3 F = ma Normal to a Streamline $\sum \delta F_n = \frac{\delta m V^2}{\Re} = \frac{\rho \delta \forall V^2}{\Re}$ $\delta \vec{W}_n = -\delta \vec{W} \cos\theta = -\lambda \delta \forall \cos\theta$ $\delta F_{pn} = (p - \delta p_n)\delta s \delta y - (p + \delta p_n)\delta s \delta y = -2 p_n \delta s \delta y$ $= -\frac{\partial p}{\partial n}\delta s \delta n \delta y = -\frac{\partial p}{\partial n}\delta \forall$ $\sum \delta F_n = \delta \vec{W}_n + \delta F_{pn} = \left(-\gamma \cos\theta - \frac{\partial p}{\partial n}\right)\delta \forall$ $-\gamma \frac{dz}{dn} - \frac{\partial p}{\partial n} = \frac{\rho V^2}{\Re} \quad \frac{\partial p}{\partial n} = -\frac{\rho V^2}{\Re}$ $\left.\begin{array}{l} \left(-\gamma \frac{dz}{dn} - \frac{\partial p}{\partial n}\right)dn = \left(\frac{\rho V^2}{\Re}\right)dn \\ \frac{\partial p}{\partial n} = \frac{dp}{dn} \quad s = \text{constant} \end{array}\right\} \rightarrow$ $\int \frac{dp}{\rho} + \int \frac{V^2}{\Re} dn + gz = \text{constant across the streamline}$ $\text{p} + \rho \int \frac{\text{V}^2}{\Re} dn + \gamma z = \text{constant across the streamline}$	[four operations] (M1N3) → 1st (2A) [trigonometric functions] (MA2G2) → 10th (2F) [partial derivative] → Post-secondary [radius] (M3G1) → 3rd (2B)	[force] (S4P3) → 4th (3A) or (S8P3) → 8th (3C) [gravity] (S6E1) → 6th (3A) [mass] (S8P3) → 8th (3A) [acceleration] (S8P3) → 8th (3C)	PS	9th + PS

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code)** **[Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 3 Elementary Fluid Dynamics – The Bernoulli Equation (Continued)				
3.4 Physical Interpretation $p + \frac{1}{2}\rho V^2 + \gamma z = \text{Constant along the streamline}$ $p + \rho \int \frac{V^2}{\Re} dn + \gamma z = \text{constant across the streamline}$ $\frac{p}{\gamma} + \frac{V^2}{2g} + z = \text{constant on a streamline}$	[four operations] (M1N3) → 1st (2A) [integration] → 12th (To be taught) Note: The main formula $p + \frac{1}{2}\rho V^2 + \gamma z = \text{Constant along the streamline}$ $\frac{p}{\gamma} + \frac{V^2}{2g} + z = \text{constant on a streamline}$ do not need calculus	[density] (S6E5) → 6th (4A) [speed] (S2P3) → 2nd (3A) [gravity] (S6E1) → 6th (3A)	9th	9th + PS
3.5 Static, Stagnation, Dynamic, and Total Pressure $p_2 = p_1 + \frac{1}{2}\rho V_1^2$ $p + \frac{1}{2}V^2 + \gamma z = p_T = \text{constant along a streamline}$ $\left.\begin{array}{l} p_3 = p + \frac{1}{2}\rho V^2 \\ p_4 = p_1 = p \end{array}\right\} \rightarrow \quad p_3 - p_4 = \frac{1}{2}\rho V^2$ $V = \sqrt{\frac{2(p_3 - p_4)}{\rho}}$	[four operations] (M1N3) → 1st (2A) [square root] (M8N1) → 8th (2A)	[pressure] (SC5) → 9th (4B) → To be taught [density] (S6E5) → 6th (4A) [speed] (S2P3) → 2nd (3A)	9th	
3.6 Examples of Use of the Bernoulli Equation $p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$	[four operations] (M1N3) → 1st (2A) [exponent] (M6A3) → 6th (2A)	[pressure] (SC5) → 9th (4B) → To be taught [density] (S6E5) → 6th (4A) [speed] (S2P3) → 2nd (3A)	9th	

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code)** **[Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 3 Elementary Fluid Dynamics – The Bernoulli Equation (Continued)				
3.6.1 Free Jets $\gamma h = \frac{1}{2}\rho V^2 \rightarrow \begin{cases} V = \sqrt{2\frac{\gamma h}{\rho}} = \sqrt{2gh} \\ V = \sqrt{2g(h+H)} \end{cases}$	[four operations] (M1N3) → 1st (2A) [exponent] (M6A3) → 6th (2A) [square root] (M8N1) → 8th (2A)	[pressure] (SC5) → 9th (4B) → To be taught [density] (S6E5) → 6th (4A) [speed] (S2P3) → 2nd (3A)	9th	9th + PS
3.6.2 Confined Flows $\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \rightarrow A_1 V_1 = A_2 V_2 \rightarrow Q_1 = Q_2$	[four operations] (M1N3) → 1st (2A) [exponent] (M6A3) → 6th (2A) [areas of geometric shapes] (M5M1) → 5th (2B)	[density] (S6E5) → 6th (4A) [speed] (S2P3) → 2nd (3A)	9th	
3.6.3 Flowrate Measurement $p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2 \quad Q = A_1 V_1 = A_2 V_2$ $Q = A_2 \sqrt{\frac{2(p_1 - p_2)}{\rho\left[1 - \left(\frac{A_2}{A_1}\right)^2\right]}}$ $p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2 \quad Q = A_1 V_1 = b V_1 z_1 = A_2 V_2 = b V_2 z_2$ $p_1 = p_2 = 0 \rightarrow Q = z_2 b \sqrt{\frac{2g(z_1 - z_2)}{1 - \left(\frac{z_2}{z_1}\right)^2}}$ $Q = C_1 H b \sqrt{2gH} = C_1 b \sqrt{2g} H^{3/2}$ $z_1 >> z_2 \rightarrow Q = z_2 b \sqrt{2g z_1}$	[four operations] (M1N3) → 1st (2A) [exponent] (M6A3) → 6th (2A) [square root] (M8N1) → 8th (2A) [areas of geometric shapes] (M5M1) → 5th (2B)	[density] (S6E5) → 6th (4A) [speed] (S2P3) → 2nd (3A) [pressure] (SC5) → 9th (4B) → To be taught [gravity] (S6E1) → 6th (3A)	9th	
3.7 The Energy Line and the Hydraulic Grade Line $\frac{p}{\gamma} + \frac{V^2}{2g} + z = \text{constant on a streamline} = H$	[four operations] (M1N3) → 1st (2A) [exponent] (M6A3) → 6th (2A) [square root] (M8N1) → 8th (2A)	[density] (S6E5) → 6th (4A) [speed] (S2P3) → 2nd (3A) [gravity] (S6E1) → 6th (3A)	9th	

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 3 Elementary Fluid Dynamics – The Bernoulli Equation (Continued)				
3.8 Restrictions on Use of the Bernoulli Equation **3.8.1 Compressibility Effects** $RT\int\frac{dp}{p}+\frac{1}{2}V^2+gz=\text{constant} \quad \rho=\frac{p}{RT}$ $\frac{V_1^2}{2g}+z_1+\frac{RT}{g}\ln\left(\frac{p_1}{p_2}\right)=\frac{V_2^2}{2g}+z_2$ $C^{1/k}\int p^{-1/k}dp+\frac{1}{2}V^2+gz=\text{constant}$ $C^{1/k}\int_{p_1}^{p_2}p^{-1/k}dp=C^{1/k}\left(\frac{k}{k-1}\right)\left[p_2^{(k-1)/k}-p_1^{(k-1)/k}\right]$ $=\left(\frac{k}{k-1}\right)\left(\frac{p_2}{\rho_2}-\frac{p_1}{\rho_1}\right)$ $\left(\frac{k}{k-1}\right)\frac{p_1}{\rho_1}+\frac{V_1^2}{2}+gz_1=\left(\frac{k}{k-1}\right)\frac{p_2}{\rho_2}+\frac{V_2^2}{2}+gz_2$ $\frac{p_2-p_1}{p_1}=\left[\left(1+\frac{k-1}{2}Ma_1^2\right)^{k/k-1}-1\right]$ (compressible) $\left.\begin{matrix}\frac{p_2}{p_1}=\frac{V_1^2}{2RT_1}\\ Ma_1=\frac{V_1}{\sqrt{kRT_1}}\end{matrix}\right\}\rightarrow\frac{p_2-p_1}{p_1}=\frac{kMa_1^2}{2}$ (incompressible) $\frac{p_2-p_1}{p_1}=\frac{kMa_1^2}{2}\left(1+\frac{1}{4}Ma_1^2+\frac{2-k}{24}Ma_1^4+...\right)$ (compressible)	[four operations] (M1N3) → 1st (2A) [exponent] (M6A3) → 6th (2A) [logarithmic functions] (MA2A5) → 10th (2E) (To be taught as a special skill) [integration] → 12th (To be taught as a special skill) Note: The main formulas $RT\int\frac{dp}{p}+\frac{1}{2}V^2+gz=\text{constant} \quad \rho=\frac{p}{RT}$ And others do not need calculus	[density] (S6E5) → 6th (4A) [speed] (S2P3) → 2nd (3A) [gravity] (S6E1) → 6th (3A)	9th	9th + PS

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code)** **[Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 3 Elementary Fluid Dynamics – The Bernoulli Equation (Continued)				
3.8.2 Unsteady Effects $\rho \frac{\partial V}{\partial t} ds + dp + \frac{1}{2}\rho d\left(V^2\right) + \gamma\ z = 0$ (along a streamline) $p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = \rho \int_{s_1}^{s_2} \frac{\partial V}{\partial t} ds + p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$ (along a streamline)	[four operations] (M1N3) → 1st (2A) [derivative] → 12th (To be taught) [integration] → 12th (To be taught)	[pressure] (SC5) → 9th (4B) → To be taught [density] (S6E5) → 6th (4A) [speed] (S2P3) → 2nd (3A)	PS	9th + PS
3. 8.3 Rotational Effects $p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2 = \text{constant} = \mathrm{C}_{12}$ $\left.\begin{matrix} V_1 = V_2 = V_0 \\ z_1 = z_2 = 0 \\ p_1 = p_2 = p_0 \end{matrix}\right\} \rightarrow \mathrm{C}_{12} = \frac{1}{2}\rho V_0^2 + p_0$ $\left.\begin{matrix} V_3 = V_4 = V_0 \\ z_3 = z_4 = h \\ \vec{F} = m\vec{a} \\ p_3 = p_1 - \gamma h \\ p_3 = p_4 \end{matrix}\right\} \rightarrow \mathrm{C}_{34} = \mathrm{C}_{12} \rightarrow$ $p + \frac{1}{2}\rho V^2 + \gamma z = \text{constant throughout flow}$ $p_4 = p_5 + \gamma H = \gamma H \quad H = \frac{p_4}{\gamma}$ **3.8.4 Other Restrictions** **3.9 Chapter Summary and Study Guide**	[four operations] (M1N3) → 1st (2A)	[pressure] (SC5) → 9th (4B) → To be taught [density] (S6E5) → 6th (4A) [speed] (S2P3) → 2nd (3A)	9th	

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 4 Fluid Kinematics				
4.1 The Velocity Field $\vec{V} = u(x,y,z,t)\hat{i} + v(x,y,z,t)\hat{j} + w(x,y,z,t)\hat{k}$ $V = \lvert\vec{V}\rvert = (u^2 + v^2 + w^2)^{1/2}$	[four operations] (M1N3) → 1st (2A) [coordinate system] (M4G3) → 4th (2B) [functions] (MA1A1) → 9th (2E) and others → Post-secondary	[velocity] (S8P3)→ 8th (3A)	PS	PS
4.1.1 Eulerian and Lagrangian Flow Descriptions $\left.\begin{matrix} x = x_0 \\ y = y_0 \\ z = z_0 \end{matrix}\right\} \begin{matrix} T = T(x_0, y_0, z_0, t) \\ T = T(x,y,z,t) \end{matrix}$	[four operations] (M1N3) → 1st (2A) [calculus] → Post-secondary [Eulerian method] → Post-secondary [Lagrangian method] → Post-secondary	[temperature] (SP3) → 9th (3B)	PS	
4.1.2 one-, Two-, and three-Dimensional Flows $\vec{V} = \vec{V}(x,t) = u\hat{i}$ $\vec{V} = \vec{V}(x,y,,t) = u\hat{i} + v\hat{j}$ $\vec{V} = \vec{V}(x,y,z,t) = u\hat{i} + v\hat{j} + w\hat{k}$	[four operations] (M1N3) → 1st (2A) [coordinate system] (M4G3) → 4th (2B)	[velocity] (S8P3)→ 8th (3A)	PS	
4.1.3 Steady and Unsteady Flows $\frac{\partial \vec{V}}{\partial t} = 0$	[four operations] (M1N3) → 1st (2A) [partial derivative] → Post-secondary		PS	
4.1.4 Streamilnes, Streaklines, and Pathlines $\frac{dy}{dx} = \frac{v}{u}$	[four operations] (M1N3) → 1st (2A) [derivative] → 12th (To be taught)	[velocity] (S8P3)→ 8th (3A)		
4.2 The Acceleration Field $\vec{a} = \vec{a}(t)$	[four operations] (M1N3) → 1st (2A) [functions] (MA1A1) → 9th (2E) and others → Post-secondary	[velocity] (S8P3)→ 8th (3A) [acceleration] (S8P3) → 8th (3C)	PS	

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code)** **[Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 4 Fluid Kinematics (Continued)				
4.2.1 The Material Derivative $\vec{V}_A = \vec{V}_A(r_A, t) = \vec{V}_A[x_A(t), y_A(t), z_A(t), t]$ $x_A = x_A(t) \quad y_A = y_A(t) \quad z_A = z_A(t)$ $\vec{a}_A(t) = \frac{d\vec{V}_A}{dt} = \frac{\partial \vec{V}_A}{\partial t} + \frac{\partial \vec{V}_A}{\partial x}\frac{dx_A}{dt} + \frac{\partial \vec{V}_A}{\partial y}\frac{dy_A}{dt} + \frac{\partial \vec{V}_A}{\partial z}\frac{dz_A}{dt}$ $\vec{a}_A = \frac{\partial \vec{V}_A}{\partial t} + u_A\frac{\partial \vec{V}_A}{\partial x} + v_A\frac{\partial \vec{V}_A}{\partial y} + w_A\frac{\partial \vec{V}_A}{\partial z}$ $\vec{a} = \frac{\partial \vec{V}}{\partial t} + u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}$ $\left.\begin{array}{l} a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} \\ a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} \\ a_x = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} \end{array}\right\} \rightarrow \vec{a} = \frac{D\vec{V}}{Dt} \rightarrow$ $\frac{D(\ )}{Dt} \equiv \frac{\partial(\ )}{\partial t} + u\frac{\partial(\ )}{\partial x} + v\frac{\partial(\ )}{\partial y} + w\frac{\partial(\ )}{\partial z} = \frac{\partial(\ )}{\partial t} + (\vec{V}\cdot\nabla)(\ )$ (Material Derivative or Substantial Derivative) $\uparrow \left\{\begin{array}{l} \nabla(\ ) = \frac{\partial(\ )}{\partial x}\hat{i} + \frac{\partial(\ )}{\partial y}\hat{j} + \frac{\partial(\ )}{\partial z}\hat{k} \\ \vec{V}\cdot\nabla(\ ) = u\frac{\partial(\ )}{\partial x} + v\frac{\partial(\ )}{\partial y} + w\frac{\partial(\ )}{\partial z} \end{array}\right. \leftarrow \left\{\begin{array}{l} T = T(x,y,z,t) \\ \vec{V} = \vec{V}(x,y,z,t) \end{array}\right\}$ $\rightarrow \left\{\begin{array}{l} \frac{dT_A}{dt} = \frac{\partial T_A}{\partial t} + \frac{\partial T_A}{\partial x}\frac{dx_A}{dt} + \frac{\partial T_A}{\partial y}\frac{dy_A}{dt} + \frac{\partial T_A}{\partial z}\frac{dz_A}{dt} \\ \frac{DT}{Dt} = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \frac{\partial T}{\partial t} + \vec{V}\cdot\nabla T \end{array}\right.$	[four operations] (M1N3) → 1st (2A) [dot product] → To be taught as a special math topic [gradient: "del"] → Post-secondary [partial derivative] → Post-secondary	[velocity] (S8P3)→ 8th (3A) [acceleration] (S8P3) → 8th (3C)	PS	PS

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 4 Fluid Kinematics (Continued)				
4.2.2 Unsteady Effects $\because \left.\begin{matrix} \frac{\partial u}{\partial x}=0 \\ v=w=0 \end{matrix}\right\} \quad \therefore \quad \vec{a}=\frac{\partial \vec{V}}{\partial t}+u\frac{\partial \vec{V}}{\partial x}+v\frac{\partial \vec{V}}{\partial y}w\frac{\partial \vec{V}}{\partial z}=\frac{\partial \vec{V}}{\partial t}=\frac{\partial V_0}{\partial t}\hat{i}$	[four operations] (M1N3) → 1st (2A) [partial derivatives] → Post-secondary	[velocity] (S8P3)→ 8th (3A)	PS	9th + PS
4.2.3 Convective Effects $\nabla(\ )=\frac{\partial(\ )}{\partial x}\hat{i}+\frac{\partial(\ )}{\partial y}\hat{j}+\frac{\partial(\ )}{\partial z}\hat{k}$ $(\vec{V}\cdot\nabla)\vec{V}$ (Convective Acceleration)	[four operations] (M1N3) → 1st (2A) [absolute value] (M7N1) → 7th (2A) [coordinate system] (M4G3) → 4th (2B) [analytic geometry] → Post-secondary [partial derivatives] → Post-secondary	[velocity] (S8P3)→ 8th (3A) [acceleration] (S8P3) → 8th (3C)	PS	
4.2.4 Streamline Coordinates $\vec{V}=V\hat{s}$ $\vec{a}=\frac{D\vec{V}}{Dt}=a_s\hat{s}+a_n\hat{n} \quad \vec{a}=\frac{D(V\hat{s})}{Dt}=\frac{DV}{Dt}\hat{s}+V\frac{D\hat{s}}{Dt}$ $\vec{a}=\left(\frac{\partial V}{\partial t}+\frac{\partial V}{\partial s}\frac{ds}{dt}+\frac{\partial V}{\partial n}\frac{dn}{dt}\right)\hat{s}+V\left(\frac{\partial \hat{s}}{\partial t}+\frac{\partial \hat{s}}{\partial s}\frac{ds}{dt}+\frac{\partial \hat{s}}{\partial n}\frac{dn}{dt}\right)$ $\vec{a}=\left(V\frac{\partial V}{\partial s}\right)\hat{s}+V\left(V\frac{\partial \hat{s}}{\partial s}\right)$	[four operations] (M1N3) → 1st (2A) [radius] (M3G1) → 3rd (2B) [absolute value] (M7N1) → 7th (2A) [analytic geometry] → Post-secondary [partial derivative] → Post-secondary [limit] → Post-secondary	[velocity] (S8P3)→ 8th (3A) [acceleration] (S8P3) → 8th (3C)	PS	

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 4 Fluid Kinematics (Continued)				
4.2.4 Streamline Coordinates (Continued) $\left.\begin{array}{l} \delta s \to 0 \\ \lvert\hat{s}\rvert = 1 \\ \frac{\partial s}{\Re} = \frac{\lvert\delta\hat{s}\rvert}{\lvert\hat{s}\rvert} = \lvert\delta\hat{s}\rvert \\ \left\lvert\frac{\lvert\delta\hat{s}\rvert}{\lvert\delta s\rvert} = \frac{1}{\Re}\right\rvert \end{array}\right\} \rightarrow \frac{\partial\hat{s}}{\partial s} = \lim_{\delta s\to 0}\frac{\delta\hat{s}}{\delta s} = \frac{\hat{n}}{\Re}$ $\vec{a} = V\frac{\partial V}{\partial s}\hat{s} + \frac{V^2}{\Re}\hat{n} \leftarrow \begin{cases} a_s = \frac{\partial V}{\partial s} \\ a_n = \frac{V^2}{\Re} \end{cases}$	[four operations] (M1N3) → 1st (2A) [radius] (M3G1) → 3rd (2B) [absolute value] (M7N1) → 7th (2A) [analytic geometry] → 12th (To be taught) [partial derivative] → Post-secondary [limit] → Post-secondary	[velocity] (S8P3)→ 8th (3A) [acceleration] (S8P3) → 8th (3C)	PS	9th + PS
4.3 Control Volume and System Representations $F = \frac{d(mv)}{dt}$	[four operations] (M1N3) → 1st (2A) [volume] (M5M4) → 5th (1B) (M6M3) → 6th (2B) (MA1G5) → 9th (2F) [areas of geometric shapes] (M5M1) → 5th (2B)	[velocity] (S8P3)→ 8th (3A)	9th	
4.4 The Reynolds Transport Theorem $B = mb \begin{cases} B = m \rightarrow b = 1 \\ B = \frac{mV^2}{2} \rightarrow b = \frac{V^2}{2} \\ \vec{B} = m\vec{V} \rightarrow \vec{b} = \vec{V} \end{cases}$ B : Extensive Property b : Intensive Property Infinitesimal fluid particles : $\delta\forall \to 0$	[four operations] (M1N3) → 1st (2A) [integration] → 12th (To be taught)	[velocity] (S8P3)→ 8th (3A) [acceleration] (S8P3) → 8th (3C) [mass] (S8P3) → 8th (3A) [temperature] (SP3) → 9th (3B) [momentum] (SP3) → (3B)	9th	

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code)** **[Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 4 Fluid Kinematics (Continued)				
4.4 The Reynolds Transport Theorem (Continued) $B_{sys} = \lim_{\delta\forall \to 0} = \sum_i b_i (\rho_i \delta\forall_i) = \int_{sys} \rho b \, d\forall$ $\uparrow \quad \delta B = b\rho \, \delta\forall$ $\frac{dB_{sys}}{dt} = \frac{d\left(\int_{sys} \rho b \, d\forall\right)}{dt} \quad \frac{dB_{cv}}{dt} = \frac{d\left(\int_{cv} \rho b \, d\forall\right)}{dt}$	[four operations] (M1N3) → 1st (2A) [integration] → 12th (To be taught)	[velocity] (S8P3)→ 8th (3A) [acceleration] (S8P3) → 8th (3C) [mass] (S8P3) → 8th (3A) [temperature] (SP3) → 9th (3B) [momentum] (SP3) → (3B)	PS	9th + PS
4.4.1 Derivation of the Reynolds Transport Theorem $\frac{DB_{system}}{Dt} = \frac{\partial B_{CV}}{\partial t} + \dot{m}_{out} \cdot b_{out} - \dot{m}_{in} \cdot b_{in}$ $\vec{F}_{system} = \frac{d[m\vec{V}]}{dt} = \frac{\partial [m\vec{V}]_{CV}}{\partial t} + \sum(\dot{m}_{out} \cdot \vec{V}_{out}) - \sum(\dot{m}_{in} \cdot \vec{V}_{in})$ $\leftarrow \begin{cases} \frac{d[m\vec{V}]}{dt} = m\frac{d[\vec{V}]}{dt} = m\vec{a} = \vec{F} \text{ (NewtonsSecond Law)} \\ \because \quad \dot{m} \cdot \vec{V} \equiv \frac{dm}{dt}\vec{V} = m\frac{d\vec{V}}{dt} = m\vec{a} = \vec{F} \quad \& \quad \frac{d[\vec{M}]}{dt} = \frac{d[m\vec{V}]}{dt} = m\frac{d[\vec{V}]}{dt} = m\vec{a} = \vec{F} \\ \therefore \quad \sum(\dot{m}_{out} \cdot \vec{V}_{out}) - \sum(\dot{m}_{in} \cdot \vec{V}_{in}) = \text{Momentum} \rightarrow \text{Force} \end{cases}$ Note: Other Formulas used to derive the Reynolds Transport Theorem are available in pages 171-177. $\frac{DB_{sys}}{Dt} = \frac{\partial}{\partial t}\int_{cv} \rho b \, d\forall + \int_{cs} \rho b \, \vec{V} \cdot \hat{n} \, dA$	[four operations] (M1N3) → 1st (2A) [areas of geometric shapes] (M5M1) → 5th (2B) [volume] (M5M4) → 5th (1B) (M6M3) → 6th (2B) (MA1G5) → 9th (2F) [dot product] → To be taught as a special math topic [analytic geometry] → 12th (To be taught) [integration] → 12th (To be taught)	[velocity] (S8P3)→ 8th (3A) [acceleration] (S8P3) → 8th (3C) [mass] (S8P3) → 8th (3A) [density] (S6E5) → 6th (4A) [temperature] (SP3) → 9th (3B) [momentum] (SP3) → (3B)	PS	
4.4.2 Physical Interpretation N/A				

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 4 Fluid Kinematics (Continued)				
4.4.3 Relationship to Material Derivative $\vec{V}\cdot\nabla(\ )=\frac{\partial(\ )}{\partial t}+u\frac{\partial(\ )}{\partial x}+v\frac{\partial(\ )}{\partial y}+w\frac{\partial(\ )}{\partial z}$ $\frac{D(\ )}{Dt}\equiv\frac{\partial(\ )}{\partial t}+u\frac{\partial(\ )}{\partial x}+v\frac{\partial(\ )}{\partial y}+w\frac{\partial(\ )}{\partial z}=\frac{\partial(\ )}{\partial t}+(\vec{V}\cdot\nabla)(\ )$ (Material Derivative or Substantial Derivative)	[four operations] (M1N3) → 1st (2A) [dot product] → To be taught as a special math topic [analytic geometry] → 12th (To be taught) [partial derivative] → Post-secondary	[velocity] (S8P3)→ 8th (3A) [acceleration] (S8P3) → 8th (3C)	PS	9th + PS
4.4.4 Steady Effects $\frac{DB_{sys}}{Dt}=\int_{sys}\rho b\vec{V}\cdot\hat{n}\ dA$	[four operations] (M1N3) → 1st (2A) [analytic geometry] → 12th (To be taught) [dot product] → To be taught as a special math topic [partial derivative] → Post-secondary	[velocity] (S8P3)→ 8th (3A) [density] (S6E5) → 6th (4A) [acceleration] (S8P3) → 8th (3C)	PS	
4.4.5 Unsteady Effects $\frac{DB_{sys}}{Dt}=\frac{\partial}{\partial t}\int_{cv}\rho b\ d\forall+\int_{cs}\rho b\ \vec{V}\cdot\hat{n}\ dA$ $\int_{cs}\rho b\ \vec{V}\cdot\hat{n}\ dA=0 \quad\rightarrow\quad \frac{DB_{sys}}{Dt}=\frac{\partial}{\partial t}\int_{cv}\rho b\ d\forall$ $\vec{V}=V_0\hat{i}\quad \Delta\rho=0\quad \vec{B}=\text{system momentum}=m\vec{V}=mV_0\hat{i}$ $\vec{b}=\frac{\vec{B}}{m}=\vec{V}=V_0\hat{i}\quad \begin{cases}\vec{V}\cdot\hat{n}>0 & \text{(out flow)}\\ \vec{V}\cdot\hat{n}<0 & \text{(in flow)}\\ \vec{V}\cdot\hat{n}=0 & \text{(along the side of the CV)}\end{cases}$	[four operations] (M1N3) → 1st (2A) [analytic geometry] → 12th (To be taught) [dot product] → To be taught as a special math topic [integration] → 12th (To be taught) [partial derivative] → Post-secondary	[velocity] (S8P3)→ 8th (3A) [density] (S6E5) → 6th (4A) [acceleration] (S8P3) → 8th (3C) [momentum] (SP3) → (3B)	PS	

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code)** **[Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 4 Fluid Kinematics (Continued)				
4.4.5 Unsteady Effects (Continued) $\frac{DB_{sys}}{Dt} = \frac{\partial}{\partial t}\int_{cv} \rho b\, d\forall + \int_{cs} \rho b\, \vec{V}\cdot\hat{n}\, dA$ $\int_{cs}\rho b\, \vec{V}\cdot\hat{n}\, dA = 0 \quad \rightarrow \quad \frac{DB_{sys}}{Dt} = \frac{\partial}{\partial t}\int_{cv}\rho b\, d\forall$ $\vec{V} = V_0\hat{i} \quad \Delta\rho = 0 \quad \vec{B} = \text{system momentum} = m\vec{V} = mV_0\hat{i}$ $\vec{b} = \frac{\vec{B}}{m} = \vec{V} = V_0\hat{i} \quad \left\{\begin{array}{ll} \vec{V}\cdot\hat{n} > 0 & \text{(out flow)} \\ \vec{V}\cdot\hat{n} < 0 & \text{(in flow)} \\ \vec{V}\cdot\hat{n} = 0 & \text{(along the side of the CV)} \end{array}\right\}$ $\left.\begin{array}{ll} \vec{V}\cdot\hat{n} = -V_0 & \text{(one section)} \\ \vec{V}\cdot\hat{n} = V_0 & \text{(another section)} \end{array}\right\} \rightarrow$ $\int_{cs}\rho b\vec{V}\cdot\hat{n}\, dA = \int_{cs}\rho(V_0\hat{i})(\vec{V}\cdot\hat{n})dA$ $= \int_{(1)}\rho(V_0\hat{i})(-V_0)dA + \int_{(2)}\rho(V_0\hat{i})(V_0)dA$ $= -\rho V_0^2 A_1\hat{i} + \rho V_0^2 A_1\hat{i} = 0$	[four operations] (M1N3) → 1st (2A) [analytic geometry] → 12th (To be taught) [dot product] → To be taught as a special math topic → To be taught as a special math topic [integration] → 12th (To be taught) [partial derivative] → Post-secondary	[velocity] (S8P3)→ 8th (3A) [density] (S6E5) → 6th (4A) [acceleration] (S8P3) → 8th (3C) [momentum] (SP3) → (3B)	PS	9th + PS
4.4.6 Moving Control Volumes $\vec{V}_{cv} = \vec{V} - \vec{W} \quad \vec{V} = \vec{W} + \vec{V}_{cv}$ $\frac{DB_{sys}}{Dt} = \frac{\partial}{\partial t}\int_{cv}\rho b\, d\forall + \int_{cs}\rho b\vec{W}\cdot\hat{n}\, dA$	[four operations] (M1N3) → 1st (2A) [analytic geometry] [integration] → 12th (To be taught) [partial derivative] → Post-secondary	[velocity] (S8P3)→ 8th (3A) [density] (S6E5) → 6th (4A) [acceleration] (S8P3) → 8th (3C) [momentum] (SP3) → (3B)	PS	

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code)** **[Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 4 Fluid Kinematics (Continued)				
4.4.7 Selection of a Control Volume N/A	N/A	N/A	9th	9th + PS
4.5 Chapter Summary and study Guide N/A	N/A	N/A	9th	
Chapter 5 Finite Control Volume Analysis				
5.1 Conservation of Mass – The Continuity Equation **5.1.1 Derivation of the Continuity Equation** $\frac{DM_{sys}}{Dt}=0 \quad M_{sys}=\int_{sys}\rho\, d\forall$ $\frac{D}{Dt}\int_{sys}\rho\, D\forall=\frac{\partial}{\partial t}\int_{cv}\rho\, d\forall+\int_{cv}\rho\vec{V}\cdot\hat{n}\, dA \quad \frac{\partial}{\partial t}\int_{cv}\rho\, d\forall \quad \int_{cs}\rho\vec{V}\cdot\hat{n}\, dA$ $\frac{\partial}{\partial t}\int_{cv}\rho\, d\forall=0 \quad \int_{cs}\rho\vec{V}\cdot\hat{n}\, dA$ $\int_{cs}\rho\vec{V}\cdot\hat{n}\, dA=\sum\dot{m}_{out}-\sum\dot{m}_{in} \quad \frac{\partial}{\partial t}\int_{cv}\rho\, \forall+\int_{cs}\rho\forall\cdot\hat{n}\, dA=0$ $\dot{m}=\rho Q=\rho AV \quad \dot{m}=\int_{A}\rho\vec{V}\cdot\hat{n}\, dA$ $V_{average}=\bar{V}=\frac{\int_{A}\rho\vec{V}\cdot\hat{n}\, dA}{\rho A}$ $V_{average}=\bar{V}=\frac{\int_{A}\rho\vec{V}\cdot\hat{n}\, dA}{\rho A}=V$ For uniformly distributed velocity (one-dimensional flow)	[four operations] (M1N3) → 1st (2A) [analytic geometry] → 12th (To be taught) [volume] (M5M4) → 5th (1B) (M6M3) → 6th (2B) (MA1G5) → 9th (2F) [areas of geometric shapes] (M5M1) → 5th (2B) [integration] → 12th (To be taught as a special skill)	[mass] (S8P3) → 8th (3A) [density] (S6E5) → 6th (4A) [velocity] (S8P3)→ 8th (3A)	PS	9th + PS

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code)** [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 5 Finite Control Volume Analysis (Continued)				
5.1.2 Fixed, Non-deforming Control Volume $\frac{\partial}{\partial t}\int_{cv}\rho\, d\forall \quad \sum \dot{m}_{out} - \sum \dot{m}_{in} = 0 \quad \sum Q_{out} - \sum Q_{in} = 0$ $\frac{\partial}{\partial t}\int_{cv}\rho\, d\forall$ $\dot{m} = \rho A V$ uniformly distributed over the opening in the control surface (one-dimensional flow) $\dot{m} = \rho A \overline{V}$ uniformly distributed over the opening in the control surface $\dot{m} = \rho_1 A_1 \overline{V}_1 = \rho_2 A_2 \overline{V}_2 \quad Q = A_1 \overline{V}_1 = A_2 \overline{V}_2$ $\sum \dot{m}_{in} = \sum \dot{m}_{out}$	[four operations] (M1N3) → 1st (2A) [analytic geometry] → 12th (To be taught) [volume] (M5M4) → 5th (1B) (M6M3) → 6th (2B) (MA1G5) → 9th (2F) [areas of geometric shapes] (M5M1) → 5th (2B) [integration] → 12th (To be taught as a special skill) [sigma notation] (M6N1) → 6th (1A) or (MA1A3) → 9th (2E) Note: The main formula $\dot{m} = \rho_1 A_1 \overline{V}_1 = \rho_2 A_2 \overline{V}_2 \quad Q = A_1 \overline{V}_1 = A_2 \overline{V}_2$ $\sum \dot{m}_{in} = \sum \dot{m}_{out}$ are not based on calculus	[mass] (S8P3) → 8th (3A) [density] (S6E5) → 6th (4A) [velocity] (S8P3) → 8th (3A)	9th	9th + PS
5.1.3 Moving, Non-deforming Control Volume $\vec{V} = \vec{W} + \vec{V}_{cv} \quad \frac{DM_{sys}}{Dt} = \frac{\partial}{\partial t}\int_{cv}\rho\, d\forall + \int_{cs}\rho \vec{W}\cdot\hat{n}\, dA = 0$	[four operations] (M1N3) → 1st (2A) [dot product] → To be taught as a special math topic [integration] → 12th (To be taught)	[velocity] (S8P3) → 8th (3A) [density] (S6E5) → 6th (4A)	PS	
5.1.4 Deforming Control Volume $\frac{DM_{sys}}{Dt} = \frac{\partial}{\partial t}\int_{cv}\rho\, d\forall + \int_{cs}\rho \vec{W}\cdot\hat{n}\, dA = 0 \quad \frac{\partial}{\partial t}\int_{cv}\rho\, d\forall \neq 0$ $\vec{V} = \vec{W} + \vec{V}_{cs}$	[four operations] (M1N3) → 1st (2A) [analytic geometry] → 12th (To be taught) [volume] (M5M4) → 5th (1B) (M6M3) → 6th (2B) (MA1G5) → 9th (2F) [areas of geometric shapes] (M5M1) → 5th (2B) [dot product] → To be taught as a special math topic [partial derivatives] → Post-secondary	[velocity] (S8P3) → 8th (3A) [density] (S6E5) → 6th (4A)	PS	
5.2 Newton's Second Law – The Linear Momentum and Moment-of-Momentum Equation N/A	N/A	N/A	9th	

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)** — **Math**	**Physics/Chemistry**	**Possible Grade to Start the Topic** — **Sec**	**Ch**
Chapter 5 Finite Control Volume Analysis (Continued)				
5.2.1 Derivation of the Linear Momentum Equation $\frac{\partial}{\partial}\int_{sys}\vec{V}\rho\, dV = \sum \vec{F}_{sys}$ $\sum \vec{F}_{sys} = \sum \vec{F}_{\text{contentof thecoincidentcontrolvolume}}$ $\frac{D}{Dt}\int_{sys}\vec{V}\rho\, dV = \frac{\partial}{\partial t}\int_{cv}\vec{V}\rho\, dV + \int_{cs}\vec{V}\rho\vec{V}\cdot\hat{n}\, dA$ $\frac{\partial}{\partial t}\int_{cv}\vec{V}\rho\, dV + \int_{cs}\vec{V}\rho\vec{V}\cdot\hat{n}\, dA = \sum \vec{F}_{\text{contentof the controlvolume}}$	[four operations] (M1N3) → 1st (2A) [areas of geometric shapes] (M5M1) → 5th (2B) [vector] (MA3A10) → 11th (2H) → To be taught as a special math topics [volume] (M5M4) → 5th (1B) (M6M3) → 6th (2B) (MA1G5) → 9th (2F) [dot product] → To be taught as special math topic [analytic geometry] → 12th (To be taught) [integration] → 12th (To be taught as special skill) [derivative] → 12th (To be taught) [partial derivative]	[velocity] (S8P3)→ 8th (3A) [force] (S4P3) → 4th (3A) or (S8P3) → 8th (3C) [density] (S6E5) → 6th (4A)	PS	9th + PS
5.2.2 Application of the Linear Momentum Equation $\frac{D}{Dt}\int_{sys}\vec{V}\rho\, dV = \frac{\partial}{\partial t}\int_{cv}\vec{V}\rho\, dV + \int_{cs}\vec{V}\rho\vec{W}\cdot\hat{n}\, dA$ $\frac{\partial}{\partial t}\int_{cv}\vec{V}\rho\, dV + \int_{cs}\vec{V}\rho\vec{W}\cdot\hat{n}\, dA = \sum \vec{F}_{\text{contentof the controlvolum}}$ $\frac{\partial}{\partial t}\int_{cv}(\vec{W}+\vec{V}_{cv})\rho\, dV + \int_{cs}(\vec{W}+\vec{V}_{cv})\rho\vec{W}\cdot\hat{n}\, dA = \sum \vec{F}_{\text{contentsof the controlvolume}}$ For constant control volume volocity $\frac{\partial}{\partial t}\int_{cv}(\vec{W}+\vec{V}_{cv})\rho\, dV \rightarrow$ For inertial, nondeforming control volume $\int_{cs}(\vec{W}+\vec{V}_{cv})\rho\vec{W}\cdot\hat{n}\, dA = \int_{cs}\vec{W}\rho\vec{W}\cdot\hat{n}\, dA + \vec{V}_{cv}\int_{cs}\rho\vec{W}\cdot\hat{n}\, dA$ For steady flow (on an instantaneous or time - average basis $\int_{cs}\rho\vec{W}\cdot\hat{n}\, dA = 0$ $\int_{cs}\vec{W}\rho\vec{W}\cdot\hat{n}\, dA = \sum \vec{F}_{\text{contentof the controlvolume}}$	[four operations] (M1N3) → 1st (2A) [areas of geometric shapes] (M5M1) → 5th (2B) [volume] (M5M4) → 5th (1B) (M6M3) → 6th (2B) (MA1G5) → 9th (2F) [vector] (MA3A10) → 11th (2H) → To be taught as a special math topics [dot product] → To be taught as a special math topic [analytic geometry] → 12th (To be taught) [integration] → 12th (To be taught as a special skill) [derivative] → 12th (To be taught) [partial derivative] → Post-secondary	[velocity] (S8P3)→ 8th (3A) [force] (S4P3) → 4th (3A) or (S8P3) → 8th (3C) [density] (S6E5) → 6th (4A)	PS	

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code)** **[Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 5 Finite Control Volume Analysis (Continued)				
5.2.3 Derivation of the Moment-of-Momentum Equation $\frac{D}{Dt}\left(\vec{V}\rho\delta\forall\right)=\delta\vec{F}_{\text{particle}} \quad \vec{r}\times\frac{D}{Dt}\left(\vec{V}\rho\ \delta\forall\right)=\vec{r}\times\delta\vec{F}_{\text{particle}}$ $\frac{D}{Dt}\left[\left(\vec{r}\times\vec{V}\right)\rho\delta\forall\right]=\frac{D\vec{r}}{Dt}\times\vec{V}\rho\delta\forall+\vec{r}\times\frac{D\left(\vec{V}\rho\delta\forall\right)}{Dt}$ $\frac{D\vec{r}}{Dt}=\vec{V} \quad \vec{V}\times\vec{V}=0 \quad \rightarrow \quad \frac{D}{Dt}\left[\left(\vec{r}\times\vec{V}\right)\rho\delta\forall\right]=\vec{r}\times\delta\vec{F}_{\text{particle}}$ $\int_{sys}\frac{D}{Dt}\left[\left(\vec{r}\times\vec{V}\right)\rho\ d\forall\right]=\sum\left(\vec{r}\times\vec{F}\right)_{sys}$ $\sum\vec{r}\times\delta\vec{F}_{\text{particle}}=\sum\left(\vec{r}\times\vec{F}\right)_{sys}$ $\frac{D}{Dt}\int_{sys}\left(\vec{r}\times\vec{V}\right)\rho\ d\forall=\int_{sys}\frac{D}{Dt}\left[\left(\vec{r}\times\vec{V}\right)\rho\ d\forall\right]$ $\frac{D}{Dt}\int_{sys}\left(\vec{r}\times\vec{V}\right)\rho\ d\forall=\sum\left(\vec{r}\times\vec{F}\right)_{sys}$ $\sum\left(\vec{r}\times\vec{F}\right)_{sys}=\sum\left(\vec{r}\times\vec{F}\right)_{cv}$ $\frac{D}{Dt}\int_{sys}\left(\vec{r}\times\vec{V}\right)\rho\ d\forall=\frac{\partial}{\partial t}\int_{cv}\left(\vec{r}\times\vec{V}\right)\rho\ d\forall+\int_{cs}\left(\vec{r}\times\vec{V}\right)\rho\vec{V}\cdot\hat{n}\ dA$ $\frac{\partial}{\partial t}\int_{cv}\left(\vec{r}\times\vec{V}\right)\rho\ d\forall+\int_{cs}\left(\vec{r}\times\vec{V}\right)\rho\vec{V}\cdot\hat{n}\ dA=\sum\left(\vec{r}\times\vec{F}\right)_{\substack{\text{contents of the}\\ \text{control volume}}}$	[four operations] (M1N3) → 1st (2A) [areas of geometric shapes] (M5M1) → 5th (2B) [volume] (M5M4) → 5th (1B) (M6M3) → 6th (2B) (MA1G5) → 9th (2F) [vector] (MA3A10) → 11th (2H) → To be taught as a special math topics [analytic geometry] → 12th (To be taught) [cross product] → To be taught as a special math topic [integration] → 12th (To be taught) [derivative] → 12th (To be taught) [partial derivative] → Post-secondary [sigma notation] (M6N1) → 6th (1A) or (MA1A3) → 9th (2E)	[velocity] (S8P3)→ 8th (3A) [force] (S4P3) → 4th (3A) or (S8P3) → 8th (3C) [density] (S6E5) → 6th (4A)	PS	9th + PS

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code)** **[Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 5 Finite Control Volume Analysis (Continued)				
5.2.4 Application of the Moment-of-Momentum Equation $\frac{\partial}{\partial t}\int_{cv}(\vec{r}\times\vec{V})\rho\, d\forall = 0$ $\int_{cs}(\vec{r}\times\vec{V})\rho\vec{V}\cdot\hat{n}\, dA$ $\vec{V}=\vec{W}+\vec{U}$ $\int_{cs}(\vec{r}\times\vec{V})\rho\vec{V}\cdot\hat{n}\, dA$ $\left[\int_{cs}(\vec{r}\times\vec{V})\rho\vec{V}\cdot\hat{n}\, dA\right]_{axial} = (-r_2V_{\theta 2})(+\dot{m})$ $\sum\left[(\vec{r}\times\vec{F})_{\text{contents of the control volume}}\right]_{axial} = T_{shaft}$ $-r_2V_{\theta 2}\dot{m} = T_{shaft}$ $\dot{W}_{shaft} = T_{shaft}\omega = -r_2V_{\theta 2}\dot{m}\omega$ $\dot{W}_{shaft} = -U_2V_{\theta 2}\dot{m}$ $w_{shaft} = -U_2V_{\theta 2}$ $T_{shaft} = (-\dot{m}_{in})(\pm r_{in}V_{\theta in}) + \dot{m}_{out}(\pm r_{out}V_{\theta out})$ $\dot{W}_{shaft} = T_{shaft}\omega$ $\dot{W}_{shaft} = (-\dot{m}_{in})(\pm r_{in}\omega V_{\theta in}) + \dot{m}_{out}(\pm r_{out}\omega V_{\theta out})$ $r\omega = U \rightarrow \dot{W}_{shaft} = (-\dot{m}_{in})(\pm U_{in}\omega V_{\theta in}) + \dot{m}_{out}(\pm U_{out}\omega V_{\theta out})$ $\dot{m} = \dot{m}_{in} = \dot{m}_{out}$ $w_{shaft} = -(\pm U_{in}V_{\theta in}) + (\pm U_{out}V_{\theta out})$	[four operations] (M1N3) → 1st (2A) [areas of geometric shapes] (M5M1) → 5th (2B) [volume] (M5M4) → 5th (1B) (M6M3) → 6th (2B) (MA1G5) → 9th (2F) [vector] (MA3A10) → 11th (2H) → To be taught as a special math topics [dot product] and [cross product] → To be taught as a special math topics [analytic geometry] → 12th (To be taught) [integration] → 12th (To be taught as a special skill) [derivative] → 12th (To be taught) [partial derivative] → Post-secondary	[velocity] (S8P3)→ 8th (3A) [force] (S4P3) → 4th (3A) or (S8P3) → 8th (3C) [density] (S6E5) → 6th (4A)	PS	PS
5.3 First Law of Thermodynamics – The Energy Equation N/A **5.3.1 Derivation of the Energy Equation** $\frac{D}{Dt}\int_{sys} e\rho\, d\forall = \left(\sum\dot{Q}_{in} - \sum\dot{Q}_{out}\right)_{sys} + \left(\sum\dot{W}_{in} - \sum\dot{W}_{out}\right)_{sys}$ $\frac{D}{Dt}\int_{sys} e\rho\, d\forall = \left(\dot{Q}_{\substack{net\\in}} + \dot{W}_{\substack{net\\in}}\right)_{sys}$ $e = \check{u} + \frac{V^2}{2} + gz$ $\left(\dot{Q}_{\substack{net\\in}} + \dot{W}_{\substack{net\\in}}\right)_{sys} = \left(\dot{Q}_{\substack{net\\in}} + \dot{W}_{\substack{net\\in}}\right)_{\text{coincident control volume}}$ $\frac{D}{Dt}\int_{sys} e\rho\, d\forall = \frac{\partial}{\partial t}\int_{cv} e\rho\, d\forall + \int_{cs} e\rho\vec{V}\cdot\hat{n}\, dA$	[four operations] (M1N3) → 1st (2A) [areas of geometric shapes] (M5M1) → 5th (2B) [volume] (M5M4) → 5th (1B) (M6M3) → 6th (2B) (MA1G5) → 9th (2F) [analytic geometry] → 12th (To be taught) [integration] → 12th (To be taught as a special skill) [derivative] → 12th (To be taught) [partial derivative] → Post-secondary [sigma notation] (M6N1) → 6th (1A) or (MA1A3) → 9th (2E)	[density] (S6E5) → 6th (4A) [heat] (S2P2) → 2nd (3A)	PS	

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code)** **[Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 5 Finite Control Volume Analysis (Continued)				
5.3.1 Derivation of the Energy Equation (Continued) $\frac{\partial}{\partial t}\int_{cv} e\rho\, d\forall + \int_{cs} e\rho\vec{V}\cdot\hat{n}\, dA = \left(\dot{Q}_{\underset{in}{net}} + \dot{W}_{\underset{in}{net}}\right)_{cv}$ $\dot{Q}_{\underset{in}{net}} = 0 \quad \rightarrow \quad \sum \dot{Q}_{in} - \sum \dot{Q}_{ou} = 0$ $\dot{W}_{shaft} = T_{shaft}\omega \quad \dot{W}_{\underset{\text{net in}}{\text{shaft}}} = \sum \dot{W}_{\underset{\text{in}}{\text{shaft}}} - \sum \dot{W}_{\underset{\text{out}}{\text{shaft}}}$ $\sigma = -p \quad \delta\dot{W}_{\text{normal stress}} = \delta\vec{F}_{\text{normal stress}}\cdot\vec{V}$ $\delta\dot{W}_{\text{normal stress}} = \sigma\hat{n}\,\delta A\cdot\vec{V} = -p\hat{n}\,\delta A\cdot\vec{V} = -p\vec{V}\cdot\hat{n}\,\delta A$ $\dot{W}_{\text{normal stress}} = \int_{cs}\sigma\vec{V}\cdot\hat{n}\, dA = \int_{cs} -p\vec{V}\cdot\hat{n}\, dA$ $\delta\dot{W}_{\text{tangential stress}} = \delta\vec{F}_{\text{tangential stress}}\cdot\vec{V}$ $\frac{\partial}{\partial t}\int_{cs} e\rho\, d\forall + \int_{cs} e\rho\vec{V}\cdot\hat{n}\, dA = \dot{Q}_{\underset{\text{in}}{\text{net}}} + \dot{W}_{\underset{\text{net in}}{\text{shaft}}} - \int_{cs} p\vec{V}\cdot\hat{n}\, dA$ Energy Equation: $\frac{\partial}{\partial t}\int_{cs} e\rho\, d\forall + \int_{cs}\left(\check{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz\right)\rho\vec{V}\cdot\hat{n}\, dA = \dot{Q}_{\underset{\text{in}}{\text{net}}} + \dot{W}_{\underset{\text{net in}}{\text{shaft}}}$	[four operations] (M1N3) → 1st (2A) [areas of geometric shapes] (M5M1) → 5th (2B) [volume] (M5M4) → 5th (1B) (M6M3) → 6th (2B) (MA1G5) → 9th (2F) [analytic geometry] → 12th (To be taught) [integration] → 12th (To be taught as a special skill) [derivative] → 12th (To be taught) [partial derivative] → Post-secondary [sigma notation] (M6N1) → 6th (1A) or (MA1A3) → 9th (2E)	[density] (S6E5) → 6th (4A) [heat] (S2P2) → 2nd (3A)	PS	9th + PS
5.3.2 Application of the Energy Equation $\int_{cs}\left(\check{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz\right)\rho\vec{V}\cdot\hat{n}\, dA \neq 0 \quad \leftarrow \quad \vec{V}\cdot\hat{n} \neq 0$ $\int_{cs}\left(\check{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz\right)\rho\vec{V}\cdot\hat{n}\, dA = \sum_{\underset{out}{flow}}\left(\check{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz\right)\dot{m}$ $-\sum_{\underset{in}{flow}}\left(\check{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz\right)\dot{m}$	[four operations] (M1N3) → 1st (2A) [areas of geometric shapes] (M5M1) → 5th (2B) [volume] (M5M4) → 5th (1B) (M6M3) → 6th (2B) (MA1G5) → 9th (2F) [analytic geometry] → 12th (To be taught) [integration] → 12th (To be taught as a special skill)	[velocity] (S8P3)→ 8th (3A) [gravity] (S6E1) → 6th (3A) [density] (S6E5) → 6th (4A) [mass] (S8P3) → 8th (3A)	PS	

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 5 Finite Control Volume Analysis (Continued)				
5.3.2 Application of the Energy Equation (Continued) $\int_{cs}\left(\check{u}+\frac{p}{\rho}+\frac{V^2}{2}+gz\right)\rho\vec{V}\cdot\hat{n}\,dA=\left(\check{u}+\frac{p}{\rho}+\frac{V^2}{2}+gz\right)_{out}\dot{m}_{out}-\left(\check{u}+\frac{p}{\rho}+\frac{V^2}{2}+gz\right)_{in}\dot{m}_{in}$ $\dot{m}\left[\check{u}_{out}-\check{u}_{in}+\left(\frac{p}{\rho}\right)_{out}-\left(\frac{p}{\rho}\right)_{in}+\frac{V_{out}^2-V_{in}^2}{2}+g\left(z_{out}-z_{in}\right)\right]=\dot{Q}_{\substack{net\\in}}+\dot{W}_{\substack{shaft\\net\ in}}$ $\check{h}=\check{u}+\frac{p}{\rho}\rightarrow$ $\dot{m}\left[\check{h}_{out}-\check{h}_{in}+\frac{V_{out}^2-V_{in}^2}{2}+g\left(z_{out}-z_{in}\right)\right]=\dot{Q}_{\substack{net\\in}}+\dot{W}_{\substack{shaft\\net\ in}}$ Enthalpy for steady throughout, one-dimensional flow involving only one fluid stream $\dot{m}\left[\check{u}_{out}-\check{u}_{in}+\left(\frac{p}{\rho}\right)_{out}-\left(\frac{p}{\rho}\right)_{in}+\frac{V_{out}^2-V_{in}^2}{2}+g\left(z_{out}-z_{in}\right)\right]=\dot{Q}_{\substack{net\\in}}$ Enthalpy for compressive, one-dimensional, steady flow $\dot{m}\left[\check{h}_{out}-\check{h}_{in}+\frac{V_{out}^2-V_{in}^2}{2}+g\left(z_{out}-z_{in}\right)\right]=\dot{Q}_{\substack{net\\in}}$	[four operations] (M1N3) → 1st (2A) [areas of geometric shapes] (M5M1) → 5th (2B) [volume] (M5M4) → 5th (1B) (M6M3) → 6th (2B) (MA1G5) → 9th (2F) [analytic geometry] → 12th (To be taught) [integration] → 12th (To be taught as a special skill)	[velocity] (S8P3)→ 8th (3A) [gravity] (S6E1) → 6th (3A) [density] (S6E5) → 6th (4A) [mass] (S8P3) → 8th (3A)	PS	9th + PS

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 5 Finite Control Volume Analysis (Continued)				
5.3.3 Comparison of the Energy Equation with the Bernoulli Equation $\dot{m}\left[\check{u}_{out}-\check{u}_{in}+\frac{p_{out}}{\rho}-\frac{p_{in}}{\rho}+\frac{V_{out}^2-V_{in}^2}{2}+g(z_{out}-z_{in})\right]=\dot{Q}_{\substack{net\\in}}$ $\frac{p_{out}}{\rho}+\frac{V_{out}^2}{2}+gz_{out}=\frac{p_{in}}{\rho}+\frac{V_{in}^2}{2}+gz_{in}-\left(\check{u}_{out}-\check{u}_{in}-q_{\substack{net\\in}}\right)$ $q_{\substack{net\\in}}=\frac{\dot{Q}_{net\ in}}{\dot{m}}$ $p_{out}+\frac{\rho V_{out}^2}{2}+\gamma z_{out}=p_{in}+\frac{\rho V_{in}^2}{2}+\gamma z_{in} \quad \gamma=\rho g \rightarrow \frac{\gamma}{\rho}=g$ $\frac{\left(p_{out}+\frac{\rho V_{out}^2}{2}+\gamma z_{out}\right)}{\rho}=\frac{\left(p_{in}+\frac{\rho V_{in}^2}{2}+\gamma z_{in}\right)}{\rho} \rightarrow$ $\frac{p_{out}}{\rho}+\frac{V_{out}^2}{2}+gz_{out}=\frac{p_{in}}{\rho}+\frac{V_{in}^2}{2}+gz_{in}$ $\check{u}_{out}-\check{u}_{in}-q_{\substack{net\\in}}=0$ (Frictionless steady incompressible flow) $\check{u}_{out}-\check{u}_{in}-q_{\substack{net\\in}}>0$ (Steady incompressible flow with friction) $\check{u}_{out}-\check{u}_{in}-q_{\substack{net\\in}}=\text{loss}$	[four operations] (M1N3) → 1st (2A) [areas of geometric shapes] (M5M1) → 5th (2B) [volume] (M5M4) → 5th (1B) (M6M3) → 6th (2B) (MA1G5) → 9th (2F)	[velocity] (S8P3)→ 8th (3A) [gravity] (S6E1) → 6th (3A) [density] (S6E5) → 6th (4A) [mass] (S8P3) → 8th (3A)	9th	9th + PS

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code)** **[Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 5 Finite Control Volume Analysis (Continued)				
5.3.3 Comparison of the Energy Equation with the Bernoulli Equation $\frac{p_{out}}{\rho}+\frac{V_{out}^2}{2}+gz_{out}=\frac{p_{in}}{\rho}+\frac{V_{in}^2}{2}+gz_{in}-\text{loss}$ $\dot{m}\left[\check{u}_{out}-\check{u}_{in}+\frac{p_{out}}{\rho}-\frac{p_{in}}{\rho}+\frac{V_{out}^2-V_{in}^2}{2}+g\left(z_{out}-z_{in}\right)\right]=\dot{Q}_{\substack{net\\in}}+\dot{W}_{\substack{shaft\\net\ in}}$ $\frac{p_{out}}{\rho}+\frac{V_{out}^2}{2}+gz_{out}=\frac{p_{in}}{\rho}+\frac{V_{in}^2}{2}+gz_{in}+w_{\substack{shaft\\net\ in}}-\left(\check{u}_{out}-\check{u}_{in}-q_{\substack{net\\in}}\right)$ $\frac{p_{out}}{\rho}+\frac{V_{out}^2}{2}+gz_{out}=\frac{p_{in}}{\rho}+\frac{V_{in}^2}{2}+gz_{in}+w_{\substack{shaft\\net\ in}}-\text{loss}$ $p_{out}+\frac{\rho V_{out}^2}{2}+\gamma z_{out}=p_{in}+\frac{\rho V_{in}^2}{2}+\gamma z_{in}+\rho w_{\substack{shaft\\net\ in}}-\rho(\text{loss})$ $\frac{\left(\frac{p_{out}}{\rho}+\frac{V_{out}^2}{2}+gz_{out}=\frac{p_{in}}{\rho}+\frac{V_{in}^2}{2}+gz_{in}+w_{\substack{shaft\\net\ in}}-\text{loss}\right)}{g}\rightarrow$ $\frac{p_{out}}{\gamma}+\frac{V_{out}^2}{2g}+z_{out}=\frac{p_{in}}{\gamma}+\frac{V_{in}^2}{2g}+z_{in}+w_{\substack{shaft\\net\ in}}-h_s-h_L$ $h_s=\frac{w_{shaft\,net\ in}}{g}=\frac{\dot{W}_{\substack{shaft\\net\ in}}}{\dot{m}g}=\frac{\dot{W}_{\substack{shaft\\net\ in}}}{\gamma Q}$ $h_T=-\left(h_s+h_L\right)_T \quad h_p=\left(h_s+h_L\right)_P$	[four operations] (M1N3) → 1st (2A) [areas of geometric shapes] (M5M1) → 5th (2B) [volume] (M5M4) → 5th (1B) (M6M3) → 6th (2B) (MA1G5) → 9th (2F)	[velocity] (S8P3)→ 8th (3A) [gravity] (S6E1) → 6th (3A) [density] (S6E5) → 6th (4A) [mass] (S8P3) → 8th (3A)	9th	9th + PS

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code)** **[Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 5 Finite Control Volume Analysis (Continued)				
5.3.4 Application of the Energy Equation to Non-uniform Flow $\int_{cs} \frac{V^2}{2}\rho\vec{V}\cdot\hat{n}\;dA = \dot{m}\left(\frac{\alpha_{out}\bar{V}_{out}^2}{2} - \frac{\alpha_{in}\bar{V}_{in}^2}{2}\right)$ $\frac{\dot{m}\alpha\bar{V}^2}{2} = \int_A \frac{V^2}{2}\rho\vec{V}\cdot\hat{n}\;dA \quad \alpha = \frac{\int_A (V^2/2)\rho\vec{V}\cdot\hat{n}\;dA}{\dot{m}\alpha\bar{V}^2/2}$ $\frac{p_{out}}{\rho} + \frac{\alpha_{out}\bar{V}_{out}^2}{2} + gz_{out} = \frac{p_{in}}{\rho} + \frac{\alpha_{in}\bar{V}_{in}^2}{2} + gz_{in} + w_{\substack{shaft\\ net\ in}} - \text{loss}$ $\left(\frac{p_{out}}{\rho} + \frac{\alpha_{out}\bar{V}_{out}^2}{2} + gz_{out} = \frac{p_{in}}{\rho} + \frac{\alpha_{in}\bar{V}_{in}^2}{2} + gz_{in} + w_{\substack{shaft\\ net\ in}} - \text{loss}\right)(\rho)$ $\rightarrow$ $p_{out} + \frac{\rho\alpha_{out}\bar{V}_{out}^2}{2} + \gamma z_{out} = p_{in} + \frac{\rho\alpha_{in}\bar{V}_{in}^2}{2} + \gamma z_{in} + \rho w_{\substack{shaft\\ net\ in}} - \rho(\text{loss})$ $\frac{\left(\frac{p_{out}}{\rho} + \frac{\alpha_{out}\bar{V}_{out}^2}{2} + gz_{out} = \frac{p_{in}}{\rho} + \frac{\alpha_{in}\bar{V}_{in}^2}{2} + gz_{in} + w_{\substack{shaft\\ net\ in}} - \text{loss}\right)}{g} \rightarrow$ $\frac{p_{out}}{\gamma} + \frac{\alpha_{out}\bar{V}_{out}^2}{2g} + z_{out} = \frac{p_{in}}{\gamma} + \frac{\alpha_{in}\bar{V}_{in}^2}{2g} + z_{in} + \frac{w_{\substack{shaft\\ net\ in}}}{g} - h_L$	[four operations] (M1N3) → 1st (2A) [areas of geometric shapes] (M5M1) → 5th (2B) [volume] (M5M4) → 5th (1B) (M6M3) → 6th (2B) (MA1G5) → 9th (2F) [integration] → 12th (To be taught as a special skill) Note: The main formulas $\frac{p_{out}}{\gamma} + \frac{\alpha_{out}\bar{V}_{out}^2}{2g} + z_{out} = \frac{p_{in}}{\gamma} + \frac{\alpha_{in}\bar{V}_{in}^2}{2g} + z_{in} + \frac{w_{\substack{shaft\\ net\ in}}}{g} - h_L$ is based on pre-calculus mathematics. Others used to derive this formula could be removed from classroom instruction.	[velocity] (S8P3)→ 8th (3A) [gravity] (S6E1) → 6th (3A) [density] (S6E5) → 6th (4A) [mass] (S8P3) → 8th (3A)	9th	9th + PS
5.3.5 Combination of the Energy Equation and the Moment-of-momentum Equation $\eta = \frac{w_{\substack{shaft\\ net\ in}} - \text{loss}}{w_{\substack{shaft\\ net\ in}}}$	[four operations] (M1N3) → 1st (2A)	[heat] (S2P2) → 2nd (3A) [temperature] (SP3) → 9th (3B)	9th	

Table 2. Fluid Mechanics Topic List (Continued).

<table>
<tr><td colspan="5">Engineering Subject: Fluid</td></tr>
<tr><td rowspan="2">Engineering Analytic Topics & Typical Formulas</td><td colspan="2">Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code)
[Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)</td><td colspan="2">Possible Grade to Start the Topic</td></tr>
<tr><td>Math</td><td>Physics/Chemistry</td><td>Sec</td><td>Ch</td></tr>
<tr><td colspan="5">Chapter 5 Finite Control Volume Analysis (Continued)</td></tr>
<tr><td>5.4 Second Law of Thermodynamics – Irreversible Flow
$\check{u}_2 - \check{u}_1 - q_{\substack{net\\in}} \geq 0$</td><td>[four operations] (M1N3) → 1st (2A)</td><td>[heat] (S2P2) → 2nd (3A)
[temperature] (SP3) → 9th (3B)</td><td>PS</td><td rowspan="3">9th
+
PS</td></tr>
<tr><td>5.4.1 Semi-infinitesimal Control Volume Statement of the Energy Equation
$\underbrace{\dot{m}\left[d\,\check{u} + d\left(\frac{p}{\rho}\right) + d\left(\frac{V^2}{2}\right) + g(dz)\right] = \delta\dot{Q}_{\substack{net\\in}} \quad T\,ds = d\,\check{u} + pd\left(\frac{1}{\rho}\right)}_{\downarrow}$
$\dot{m}\left[T\,ds + pd\left(\frac{1}{\rho}\right) + d\left(\frac{p}{\rho}\right) + d\left(\frac{V^2}{2}\right) + g(dz)\right] = \delta\dot{Q}_{\substack{net\\in}}$
$\frac{dp}{\rho} + d\left(\frac{V^2}{2}\right) + g\;dz = -\left(T\,ds - \delta q_{\substack{net\\in}}\right)$</td><td>[four operations] (M1N3) → 1st (2A)
[volume] (M5M4) → 5th (1B) (M6M3) → 6th (2B) (MA1G5) → 9th (2F)
[integration] → 12th (To be taught as a special skill)
[derivative] → 12th (To be taught)
[partial derivative] → Post secondary
[sigma notation] (M6N1) → 6th (1A) or (MA1A3) → 9th (2E)</td><td>[mass] (S8P3) → 8th (3A)
[velocity] (S8P3)→ 8th (3A)
[heat] (S2P2) → 2nd (3A)
[temperature] (SP3) → 9th (3B)
[gravity] (S6E1) → 6th (3A)</td><td>PS</td></tr>
<tr><td>5.4.2 Semi-infinitesimal Control Volume Statement of the Second Law of Thermodynamics
$\frac{D}{Dt}\int_{sys} s\rho\,d\forall \geq \sum\left(\frac{\delta\dot{Q}_{\substack{net\\in}}}{T}\right)_{sys} \quad \sum\left(\frac{\delta\dot{Q}_{\substack{net\\in}}}{T}\right)_{sys} = \sum\left(\frac{\delta\dot{Q}_{\substack{net\\in}}}{T}\right)_{cv}$
$\frac{D}{Dt}\int_{sys} s\rho\,d\forall = \frac{\partial}{\partial t}\int_{cv} s\rho\,d\forall + \int_{cs} s\rho\vec{V}\cdot\hat{n}\,dA$
$\frac{\partial}{\partial t}\int_{cv} s\rho\,d\forall + \int_{cs} s\rho\vec{V}\cdot\hat{n}\,dA \geq \sum\left(\frac{\delta\dot{Q}_{\substack{net\\in}}}{T}\right)_{cv} \quad \frac{\partial}{\partial t}\int_{cv} s\rho\,d\forall = 0$
$\dot{m}\left(s_{out} - S_{IN}\right) \geq \sum\frac{\delta\dot{Q}_{\substack{net\\in}}}{T} \quad \dot{m}\;ds \geq \sum\frac{\delta\dot{Q}_{\substack{net\\in}}}{T}$
$T\,ds \geq \delta q_{\substack{net\\in}} \quad T\,ds - \delta q_{\substack{net\\in}} \geq 0$</td><td>[four operations] (M1N3) → 1st (2A)
[volume] (M5M4) → 5th (1B) (M6M3) → 6th (2B) (MA1G5) → 9th (2F)
[derivative] → 12th (To be taught)
[partial derivative] → Post secondary
[integration] → 12th (To be taught)
[sigma notation] (M6N1) → 6th (1A) or (MA1A3) → 9th (2E)</td><td>[heat] (S2P2) → 2nd (3A)
[velocity] (S8P3)→ 8th (3A)
[density] (S6E5) → 6th (4A)
[temperature] (SP3) → 9th (3B)</td><td>PS</td></tr>
</table>

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code)** **[Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 5 Finite Control Volume Analysis (Continued)				
5.4.3 Combination of the Equations of the First and Second Laws of Thermodynamics $-\left[\frac{dp}{\rho}+d\left(\frac{V^2}{2}\right)+g\ dz\right]\geq 0$ $-\left[\frac{dp}{\rho}+d\left(\frac{V^2}{2}\right)+g\ dz\right]=\delta(\text{loss})=\left(T\ ds-\delta q_{\substack{net\\in}}\right)$ $\frac{dp}{\rho}+d\left(\frac{V^2}{2}\right)+g\ dz=0$ $-\left[\frac{dp}{\rho}+d\left(\frac{V^2}{2}\right)+g\ dz\right]=\delta(\text{loss})-\delta w_{\substack{shaft\\net\ in}}$ $d\check{u}+pd\left(\frac{1}{\rho}\right)-\delta q_{\substack{net\\in}}=\delta(\text{loss})$ $d\left(\frac{1}{\rho}\right)=0\ \rightarrow\ d\check{u}-\delta q_{\substack{net\\in}}=\delta(\text{loss})$ $\check{u}_{out}-\check{u}_{in}-q_{\substack{net\\in}}=\text{loss}$ $d\left(\frac{1}{\rho}\right)\neq 0\ \rightarrow\ \check{u}_{out}-\check{u}_{in}+\int_{in}^{out}pd\left(\frac{1}{\rho}\right)-q_{\substack{net\\in}}=\text{loss}$	[four operations] (M1N3) → 1st (2A) [volume] (M5M4) → 5th (1B) (M6M3) → 6th (2B) (MA1G5) → 9th (2F) [derivative] → 12th (To be taught) [partial derivative] → Post secondary [integration] → 12th (To be taught)	[heat] (S2P2) → 2nd (3A) [velocity] (S8P3) → 8th (3A) [density] (S6E5) → 6th (4A) [temperature] (SP3) → 9th (3B)	PS	9th + PS

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 5 Finite Control Volume Analysis (Continued)				
5.4.4 Application of the Loss Form of the Energy Equation $\frac{p_2}{\rho}+\frac{V_2^2}{2}+gz_2=\frac{p_1}{\rho}+\frac{V_1^2}{2}+gz_1$ $\int_2^2\frac{dp}{\rho}+\frac{V_2^2}{2}+gz_2=\frac{V_1^2}{2}+gz_1$ $\frac{p}{\rho^k}=\text{constant}$ $\int_1^2\frac{dp}{\rho}=\frac{k}{k-1}\left(\frac{p_2}{\rho_2}-\frac{p_1}{\rho_1}\right)$ $\frac{k}{k-1}\frac{p_2}{\rho_2}+\frac{V_2^2}{2}+gz_2=\frac{k}{k-1}\frac{p_1}{\rho_1}+\frac{V_1^2}{2}+gz_1$	[four operations] (M1N3) → 1st (2A) [exponent] (M6A3) → 6th (2A) [derivative] → 12th (To be taught) [partial derivative] → Post secondary [integration] → 12th (To be taught) Note: The main formula $\frac{k}{k-1}\frac{p_2}{\rho_2}+\frac{V_2^2}{2}+gz_2=\frac{k}{k-1}\frac{p_1}{\rho_1}+\frac{V_1^2}{2}+gz_1$ is not based on calculus	[velocity] (S8P3)→ 8th (3A) [density] (S6E5) → 6th (4A) [pressure] (SC5) → 9th (4B) → To be taught [gravity] (S6E1) → 6th (3A)	9th	9th + PS
5.5 Chapter Summary and Study Guide N/A	N/A	N/A	9th	
Chapter 6 Differential Analysis of Fluid Mechanics Flow (Note: This whole Chapter appears to be too deep in calculus-based mathematics. Actually, some professors in undergraduate engineering programs cut the whole Chapter off when teaching Fluid Mechanics course. Therefore, engineering analytic principles and skills from this Chapter are NOT analyzed for the eventual inclusion into a potentially viable K-12 engineering curriculum. The subheadings of Sections are still listed below for reference purposes).				

6.1 Fluid Mechanics Element Kinematics	6.3.2 Equations of Motion	6.5.3 Vortex	6.8.2 The Navier-Stokes Equations	PS
6.1.1 Velocity and Acceleration Fields Revisited	6.4 Inviscid Flow	6.5.4 Doublet	6.9 Some Simple Solutions for Viscous, Incompressible Fluids	
6.1.2 Linear Motion and Deformation	6.4.1 Euler's Equations of Motion	6.6 Superposition of Basic, Plane Potential Flows	6.9.1 Steady, Laminar Flow between Fixed Parallel Plates	
6.1.3 Angular Motion and Deformation	6.4.2 The Bernoulli Equation	6.6.1 Source in a Uniform Stream – Half-Body	6.9.2 Couette Flow	
6.2 Conservation of mass	6.4.3 Irrotational Flow	6.6.2 Rankine Ovals	6.9.3 Steady, Laminar Flow in Circular Tubes	
6.2.1 Differential Survey Form of Continuity Equation	6.4.4 The Bernoulli Equation for Irrotational Flow	6.6.3 Flow around a Circular Cylinder	6.9.4 Steady, Axial, Laminar Flow in an Annulus	
6.2.2 Cylindrical Polar Coordinates	6.4.5 The Velocity Potential	6.7 Other Aspects of Potential Flow Analysis	6.10 Other Aspects of Differential Analysis	
6.2.3 The Stream Function	6.5 Some Basic, Plane Potential Flows	6.8 Viscous Flow	6.10.1 Numerical Methods	
6.3 Conservation of Linear Momentum	6.5.1 Uniform Flow	6.8.1 Stress-Deformation Relationships	Chapter Summary and Study Guide	
6.3.1 Description of Forces Acting on the Differential Element	6.5.2 Source and Sink			

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 7 Similitude, Dimensional Analysis, and Modeling (Note: This whole Chapter appears to be mildly deep in calculus-based mathematics. However, the type of "abstract thinking" required to understand and to apply the content knowledge contained in this Chapter appears to be most likely beyond the cognitive developmental maturity level of high school students. Therefore, engineering analytic principles and skills from this Chapter are NOT analyzed for the eventual inclusion into a potentially viable K-12 engineering curriculum. The subheadings of Sections are still listed before for reference purposes). Some appropriate skills in 7.1 (Dimensional Analysis) could be considered for high schools.				

7.1 Dimensional Analysis	7.4.3 Uniqueness of Pi Terms	7.8 Modeling and Similitude	7.9.2 Flow around Immersed Bodies	PS
7.2 Buckingham Pi Theorem	7.5 Determination of Pi Terms by Inspection	7.8.1 Theory of Models	7.9.3 Flow with a Free Surface	
7.3 Determination of Pi Terms	7.6 Common Dimensionless Groups in Fluid Mechanics	7.8.2 Model Space	7.10 Similitude Based on Governing Differential Equations	
7.4 Some Additional Comments about Dimensional Analysis	7.7 Correlation of Experimental Data	7.8.3 Practical Aspects of Using Models	7.11 Chapter Summary and Study Guide	
7.4.1 Selection of Variables	7.7.1 Problems with One Pi Term	7.9 Some Typical Model Studies		
7.4.2 Determination of Reference Dimensions	7.7.2 Problems with Two or More Pi Term	7.9.1 Flow through Closed Conduits		

Chapter 8 Viscous Flow in Pipes				
8.1 General Characteristics of Pipe Flow **8.1.1 Laminar of Turbulent Flow** $\mathrm{Re} = \frac{\rho VD}{\mu}$ **8.1.2 Entrance Region and Fully Developed Flow** $\frac{\ell_e}{D} = 0.06\,\mathrm{Re}$ (for turbulent flow) $\frac{\ell_e}{D} = 4.4(\mathrm{Re})^{1/6}$ (for turbulent flow) $10^4 < \mathrm{Re} < 10^5$	[four operations] (M1N3) → 1st (2A) [coordinate system] (M4G3) → 4th (2B) [exponent] (M6A3) → 6th (2A)	[mass] (S8P3) → 8th (3A) [density] (S6E5) → 6th (4A) [force] (S4P3) → 4th (3A) or (S8P3) → 8th (3C) [pressure] (SC5) → 9th (4B) → To be taught [velocity] (S8P3)→ 8th (3A) [momentum] (SP3) → (3B)	PS	9th + PS
8.1.3 Pressure and Shear Stress $\nabla p = p_1 - p_2 \quad \frac{\partial p}{\partial x} = -\frac{\Delta p}{\ell} < 0$	[four operations] (M1N3) → 1st (2A) [partial derivative] → Post-secondary	[pressure] (SC5) → 9th (4B) → To be taught	PS	

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 8 Viscous Flow in Pipes (Continued)				
8.2 Fully Developed Laminar Flow **8.2.1 From F = ma Applied Directly to a Fluid Mechanics Element** $\vec{F}=m\vec{a}\quad \frac{\partial \vec{V}}{\partial t}=0\quad \vec{V}\cdot\nabla\vec{V}=u\frac{\partial u}{\partial x}\hat{i}=0\quad p_2=p_1-\Delta p$ $(p_1)\pi r^2-(p_1-\Delta p)\pi r^2-(\tau)2\pi r\ell=0\quad \frac{\Delta p}{\ell}=\frac{2\tau}{r}\quad \tau=\frac{2\tau_w r}{D}$ $\Delta p=\frac{4\ell\tau_w}{D}\quad \tau=-\mu\frac{du}{dr}\quad \frac{du}{dr}=-\left(\frac{\Delta p}{2\mu\ell}\right)r\quad \int du=-\frac{\Delta p}{2\mu\ell}\int r\,dr$ $u=-\left(\frac{\Delta p}{4\mu\ell}\right)r^2+C_1\quad u(r)=\left(\frac{\Delta pD^2}{16\mu\ell}\right)\left[1-\left(\frac{2r}{D}\right)^2\right]=V_c\left[1-\left(\frac{2r}{D}\right)^2\right]$ $u(r)=\frac{\tau_w D}{4\mu}\left[1-\left(\frac{r}{R}\right)^2\right]$ $Q=\int u\,dA=\int_{r=0}^{r=R}u(r)2\pi r\,dr=2\pi V_c\int_0^R\left[1-\left(\frac{r}{R}\right)^2\right]r\,dr$ $Q=\frac{\pi R^2V_c}{2}\quad V=\frac{Q}{A}=\frac{Q}{\pi R^2}\quad V=\frac{\pi R^2V_c}{2\pi R^2}=\frac{V_c}{2}=\frac{\Delta pD^2}{32\mu\ell}$ $Q=\frac{\pi D^4\Delta p}{128\mu\ell}\quad \frac{\Delta p-\gamma\ell\sin\theta}{\ell}=\frac{2\tau}{r}\quad V=\frac{(\Delta p-\gamma\ell\sin\theta)D^2}{32\mu\ell}$ $Q=\frac{\pi(\Delta p-\gamma\ell\sin\theta)D^4}{128\mu\ell}$	[four operations] (M1N3) → 1st (2A) [coordinate system] [trigonometric functions] (MA2G2) → 10th (2F) [dot product] → To be taught as a special math topic [integration] → 12th (To be taught) [derivative] → 12th (To be taught) [partial derivative] → Post-secondary [gradient] → Post-secondary	[force] (S4P3) → 4th (3A) or (S8P3) → 8th (3C) [acceleration] (S8P3) → 8th (3C) [pressure] (SC5) → 9th (4B) → To be taught [velocity] (S8P3)→ 8th (3A)	PS	9th + PS

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code)** **[Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 8 Viscous Flow in Pipes (Continued)				
8.2.2 From the Navier-Stokes Equations $\nabla\vec{V}\cdot\vec{V}=0$ $\frac{\partial\vec{V}}{\partial t}+\vec{V}\cdot\nabla\vec{V}=-\frac{\nabla p}{\rho}+\vec{g}+\nu\nabla^2\vec{V}$ $\vec{g}=-g\hat{k}$ $\nabla\cdot\vec{V}=0$ $\nabla p+\rho g\hat{k}=\mu\nabla^2\vec{V}$ $\frac{\partial p}{\partial x}+\rho g\sin\theta=\mu\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)$	[four operations] (M1N3) → 1st (2A) [trigonometric functions] (MA2G2) → 10th (2F) [exponent] (M6A3) → 6th (2A) [dot product] → To be taught as a special math topic [vector] (MA3A10) → 11th (2H) → To be taught as a special math topics [partial derivative] → Post-secondary [gradient: "del"] → Post-secondary	[velocity] (S8P3)→ 8th (3A) [pressure] (SC5) → 9th (4B) → To be taught [density] (S6E5) → 6th (4A) [gravity] (S6E1) → 6th (3A)	PS	9th + PS
8.2.3 From Dimensional Analysis $\Delta p=F(V,\ell,D,\mu)$ $\frac{D\,\Delta p}{\mu V}=\phi\left(\frac{\ell}{D}\right)$ $\phi\left(\frac{\ell}{D}\right)=\frac{C\ell}{D}$ $C=\text{constant}$ $\frac{D\,\Delta p}{\mu V}=\frac{C\ell}{D}$ $\frac{\Delta p}{\ell}=\frac{C\mu V}{D^2}$ $Q=AV=\frac{(\pi/4C)\Delta p D^4}{\mu\ell}$ $\Delta p=\frac{32\mu\ell V}{D^2}$ $\frac{\Delta p}{\frac{1}{2}\rho V^2}=\frac{(32\mu\ell V/D^2)}{\frac{1}{2}\rho V^2}=64\left(\frac{\mu}{\rho VD}\right)\left(\frac{\ell}{D}\right)=\frac{64}{\text{Re}}\left(\frac{\ell}{D}\right)$ $\Delta p=f\frac{\ell}{D}\frac{\rho V^2}{2}$ $f=\Delta p\left(\frac{D}{\ell}\right)\left(\frac{\rho V^2}{2}\right)$ $f=\frac{64}{\text{Re}}$ $f=\frac{8\tau_w}{\rho V^2}$	[four operations] (M1N3) → 1st (2A) [exponent] (M6A3) → 6th (2A)	[force] (S4P3) → 4th (3A) or (S8P3) → 8th (3C) [velocity] (S8P3)→ 8th (3A) [pressure] (SC5) → 9th (4B) → To be taught [density] (S6E5) → 6th (4A) Note: Special topics from 7.1 (Dimensional Analysis) need to be taught	PS	
8.2.4 Energy Considerations $\frac{p_1}{\gamma}+\alpha_1\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\alpha_2\frac{V_2^2}{2g}+z_2+h_L$ $\left(\frac{p_1}{\gamma}+z_1\right)-\left(\frac{p_2}{\gamma}+z_1\right)=h_L$ $p_1=p_2+\Delta p$ $z_2-z_1=\ell\sin\theta$ $h_L=\frac{2\tau\ell}{\gamma r}$ $h_L=\frac{4\ell\tau_w}{\gamma D}$	[four operations] (M1N3) → 1st (2A) [trigonometric functions] (MA2G2) → 10th (2F)	[pressure] (SC5) → 9th (4B) → To be taught [gravity] (S6E1) → 6th (3A)	9th	

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code)** **[Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 8 Viscous Flow in Pipes (Continued)				
8.3 Fully Developed Turbulent Flow N/A	[coordinate system] (M4G3) → 4th (2B) [analytic geometry] → 12th (To be taught) [logarithmic functions] (MA2A5) → 10th (2E) (To be taught as a special skill) [integration] → 12th (To be taught as a special skill) [derivative] → 12th (To be taught)	[velocity] (S8P3)→ 8th (3A) [density] (S6E5) → 6th (4A)	PS	9th + PS
8.3.1 Transition from Laminar to Turbulent Flow N/A				
8.3.2 Turbulent Shear Stress $\bar{u} = \frac{1}{T}\int_{t_0}^{t_0+T} u(x,y,z,t)dt \quad u=\bar{u}+u' \quad u'=u-\bar{u}$ $\bar{u}' = \frac{1}{T}\int_{t_0}^{t_0+T}(u-\bar{u})dt = \frac{1}{T}\left(\int_{t_0}^{t_0+T} u\,dt - \bar{u}\int_{t_0}^{t_0+T} dt\right) = \frac{1}{T}(T\bar{u}-T\bar{u})=0$ $\overline{(u')^2} = \frac{1}{T}\int_{t_0}^{t_0+T}(u')^2 dt > 0$ Turbulence intensity $= \frac{\sqrt{\overline{(u')^2}}}{\bar{u}} = \frac{\left[\frac{1}{T}\int_{t_0}^{t_0+T}(u')^2dt\right]^{1/2}}{\bar{u}}$ $\tau=\mu\frac{du}{dy} \quad \tau\neq\mu\frac{d\bar{u}}{dy} \quad \bar{u}=\bar{u}(y) \quad \tau=\mu\frac{d\bar{u}}{dy}-\rho\overline{u'v'}=\tau_{lam}+\tau_{turb}$ $\tau_{turb}=\eta\frac{d\bar{u}}{dy} \quad \eta=\rho\ell_m^2\left\lvert\frac{d\bar{u}}{dy}\right\rvert \quad \tau_{turb}=\rho\ell_m^2\left(\frac{d\bar{u}}{dy}\right)^2$				
8.3.3 Turbulent Velocity Profile $\frac{\bar{u}}{u^*}=\frac{yu^*}{\nu} \quad y=R-r \quad u^*=\left(\frac{\tau_w}{\rho}\right)^{1/2} \quad \frac{\bar{u}}{u^*}=2.5\ln\left(\frac{yu^*}{\nu}\right)+5.0$ $\frac{(V_c-\bar{u})}{u^*}=2.5\ln\left(\frac{R}{y}\right) \quad \frac{\bar{u}}{V_c}=\left(1-\frac{r}{R}\right)^{1/n}$	[four operations] (M1N3) → 1st (2A) [coordinate system] (M4G3) → 4th (2B) [logarithmic functions] (MA2A5) → 10th (2E) [analytic geometry] → 12th (To be taught)	[velocity] (S8P3)→ 8th (3A)	PS	
8.3.4 Turbulent Modeling N/A				

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 8 Viscous Flow in Pipes (Continued)				
8.3.5 Chaos and Turbulence N/A	N/A	N/A	PS	9^{th} + PS
8.4 Dimensional Analysis of Pipe Flow $h_L = h_{L\,major} + h_{L\,minor}$	[four operations] (M1N3) → 1^{st} (2A)	Note: Special topics from 7.1 (Dimensional Analysis) need to be taught	9^{th}	
8.4.1 Major Losses $h_L = h_{L\,major} + h_{L\,minor}$ $\Delta p = F(V, D, \ell, \varepsilon, \mu, \rho)$ $\frac{\Delta p}{\frac{1}{2}\rho V^2} = \tilde{\phi}\left(\frac{\rho VD}{\mu}, \frac{\ell}{D}, \frac{\varepsilon}{D}\right)$ $\mathrm{Re} = \frac{\rho VD}{\mu}$ $\frac{\Delta p}{\frac{1}{2}\rho V^2} = \frac{\ell}{D}\phi\left(\mathrm{Re}, \frac{\varepsilon}{D}\right)$ $f = \phi\left(\mathrm{Re}, \frac{\varepsilon}{D}\right)$ $\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_L$ $h_{L\,major} = f\frac{\ell}{D}\frac{V^2}{2g}$ $p_1 - p_2 = \gamma(z_2 - z_1) + \gamma h_L = \gamma(z_2 - z_1) + f\frac{\ell}{D}\frac{\rho V^2}{2}$ $\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$	[four operations] (M1N3) → 1^{st} (2A) [areas of geometric shapes: circle, triangle] (M5M1) → 5^{th} (2B) (M5M1) → 5^{th} (2B) [logarithmic functions] (MA2A5) → 10^{th} (2E) (To be taught as a special skill) [exponent] (M6A3) → 6^{th} (2A) [square root] (M8N1) → 8^{th} (2A) [graph] (S7CS6) → 7^{th} (6)	[velocity] (S8P3)→ 8^{th} (3A) [pressure] (SC5) → 9^{th} (4B) → To be taught [force] (S4P3) → 4^{th} (3A) or (S8P3) → 8^{th} (3C) [gravity] (S6E1) → 6^{th} (3A) [density] (S6E5) → 6^{th} (4A) Note: Special topics from 7.1 (Dimensional Analysis) need to be taught	9^{th}	
8.4.2 Minor Losses $K_L = \frac{h_{L\,minor}}{V^2/2g} = \frac{\Delta p}{\frac{1}{2}\rho V^2}$ $\Delta p = K_L \frac{1}{2}\rho V^2$ $h_{L\,minor} = K_L \frac{V^2}{2g}$ $K_L = \phi(geometry, \mathrm{Re})$ $h_{L\,minor} = K_L \frac{V^2}{2g} = f\frac{\ell_{eq}}{D}\frac{V^2}{2g}$ $\ell_{eq} = \frac{K_L D}{f}$ $A_1V_1 = A_3V_3$ $p_1A_3 - p_3A_3 = \rho A_3V_3(V_3 - V_1)$ $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + h_L$ $K_L = \frac{h_L}{V_1^2/2g}$ $K_L = \left(1 - \frac{A_1}{A_2}\right)^2$ $C_p = (p_2 - p_1)\left(\frac{\rho V_1^2}{2}\right)$				

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 8 Viscous Flow in Pipes (Continued)				
8.4.3 Noncircular Conduits $f = \frac{C}{\text{Re}_h}$ $\text{Re}_h = \frac{\rho V D_h}{\mu}$ $D_h = \frac{4A}{P} = \frac{4(\pi D^2/4)}{\pi D} = D$ $h_L = f\frac{(\ell/D_h)V^2}{2g}$	[four operations] (M1N3) → 1st (2A) [exponent] (M6A3) → 6th (2A) [areas of geometric shapes] (M5M1) → 5th (2B)	[velocity] (S8P3)→ 8th (3A) [gravity] (S6E1) → 6th (3A) [density] (S6E5) → 6th (4A)	9th	9th + PS
8.5 Pipe Flow Examples N/A				
8.5.1 Single Pipes N/A				
8.5.2 Multiple Pipe Systems N/A				
8.6 Pipe Flowrate Measurement **8.6.1 Pipe Flowrate Meters** $Q_{ideal} = A_2V_2 = A_2\sqrt{\frac{2(p_1 - p_2)}{\rho(1-\beta^4)}}$ $Q = A_1V_1 = A_2V_2$ $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + h_L$ $Q = C_0 Q_{ideal} = C_0 A_0\sqrt{\frac{2(p_1 - p_2)}{\rho(1-\beta^4)}}$ $\beta = \frac{d}{D}$ $\text{Re} = \frac{\rho V D}{\mu}$ $V = \frac{Q}{A_1}$ $Q = C_n Q_{ideal} = C_n A_n\sqrt{\frac{2(p_1 - p_2)}{\rho(1-\beta^4)}}$ $Q = C_v Q_{ideal} = C_v A_T\sqrt{\frac{2(p_1 - p_2)}{\rho(1-\beta^4)}}$ **8.6.2 Volume Flow Meters** N/A	[four operations] (M1N3) → 1st (2A) [exponent] (M6A3) → 6th (2A) [square root] (M8N1) → 8th (2A) [areas of geometric shapes] (M5M1) → 5th (2B)	[velocity] (S8P3)→ 8th (3A) [pressure] (SC5) → 9th (4B) → To be taught [gravity] (S6E1) → 6th (3A) [density] (S6E5) → 6th (4A)	9th	
8.7 Chapter Summary and Study Guide N/A	N/A	N/A	9th	

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code)** **[Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 9 Flow over Immersed Bodies				
9.1 General External Flow Characteristics **9.1.1 Lift and Drag Concepts** $\left.\begin{array}{l} dF_x = (p\ dA)\cos\theta + (\tau_w\ dA)\sin\theta \\ dF_y = -(p\ dA)\sin\theta + (\tau_w\ dA)\cos\theta \end{array}\right\} \rightarrow$ $\vec{D} = \int dF_x = \int p\ \cos\theta\ dA + \int \tau_w\ \sin\theta\ dA$ $\vec{L} = \int dF_y = -\int p\ \sin\theta\ dA + \int \tau_w\ \cos\theta\ dA$ $C_L = \frac{\vec{L}}{\frac{1}{2}\rho U^2 A} \quad C_D = \frac{\vec{D}}{\frac{1}{2}\rho U^2 A}$	[four operations] (M1N3) → 1st (2A) [areas of geometric shapes] (M5M1) → 5th (2B) [trigonometric functions] (MA2G2) → 10th (2F) [integration] → 12th (To be taught as a special skill) [derivative] → 12th (To be taught) Note: The main formulas $C_L = \frac{\vec{L}}{\frac{1}{2}\rho U^2 A} \quad C_D = \frac{\vec{D}}{\frac{1}{2}\rho U^2 A}$ are not based on calculus	[force] (S4P3) → 4th (3A) or (S8P3) → 8th (3C)	9th	9th + PS
9.1.2 Characteristics of Flow Past an Object N/A	N/A	[force] (S4P3) → 4th (3A) or (S8P3) → 8th (3C) [Reynolds Number] → To be taught as special topic	9th	
9.2 Boundary Layer Characteristics N/A **9.2.1 Boundary Layer structure and Thickness on a Flat Plate** $\delta^* bU = \int_0^\infty (U-u) b\ dy \quad \delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right) dy$ $\int \rho u (U-u)\ dA = \rho b \int_0^\infty u(U-u) dy \quad \rho b U^2 \Theta = \int_0^\infty u(U-u) dy$ $\Theta = \int_0^\infty \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$	[four operations] (M1N3) → 1st (2A) [exponent] (M6A3) → 6th (2A) [areas of geometric shapes] (M5M1) → 5th (2B) [integration] → 12th (To be taught as a special skill)	[velocity] (S8P3)→ 8th (3A) [density] (S6E5) → 6th (4A)	PS	

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 9 Flow over Immersed Bodies (Continued)				
9.2.2 Prandtl/Blasius Boundary Layer Solution $u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$ $u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$ $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad \left.\begin{matrix} v<<u \\ \frac{\partial}{\partial x}<<\frac{\partial}{\partial y}\end{matrix}\right\}\rightarrow \quad \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$ $u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2} \quad u=v=0 \quad on \quad y=0$ $u\rightarrow U \quad as \quad y\rightarrow\infty \quad \frac{u}{U}=g\left(\frac{y}{\delta}\right) \quad \delta\sim\left(\frac{\nu x}{U}\right)^{1/2}$ $\eta=\left(\frac{U}{\nu x}\right)^2 \quad \Psi=(\nu xU)^{1/2}f(\eta) \quad f=f(\eta)$ $u=\frac{\partial\Psi}{\partial y} \quad v=-\frac{\partial\Psi}{\partial x} \quad u=Uf'(\eta) \quad v=\left(\frac{\nu U}{4x}\right)^{1/2}(\eta f'-f)$ $2f'''-ff''=0 \quad f=f'=0 \quad at \quad \eta=0$ $f=f'=0 \quad at \quad \eta=0 \quad and \quad f'\rightarrow 1 \quad as \quad \eta\rightarrow\infty$ $\delta=5\sqrt{\frac{\nu x}{U}} \quad \frac{\delta}{x}=\frac{5}{\sqrt{\mathrm{Re}_x}} \quad \frac{\delta^*}{x}=\frac{1.721}{\sqrt{\mathrm{Re}_x}} \quad \frac{\Theta}{x}=\frac{0.664}{\sqrt{\mathrm{Re}_x}}$ $\tau_w=0.332U^{3/2}\sqrt{\frac{\rho\mu}{x}}$	[four operations] (M1N3) → 1st (2A) [exponent] (M6A3) → 6th (2A) [square root] (M8N1) → 8th (2A) [functions] (MA1A1) → 9th (2E) and others → Post-secondary [partial derivative] → Post-secondary [3rd order non-linear differential equation] → Post-secondary	[velocity] (S8P3)→ 8th (3A) [density] (S6E5) → 6th (4A) [gravity] (S6E1) → 6th (3A)	PS	9th + PS

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code)** **[Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 9 Flow over Immersed Bodies (Continued)				
9.2.3 Momentum Integral boundary Layer Equation for a Flat Plate $\sum F_x = \rho\int_{(1)} u\vec{V}\cdot\hat{n}\ dA + \rho\int_{(2)} u\vec{V}\cdot\hat{n}\ dA$ $\sum F_x = -\vec{D} = -\int_{plate}\tau_w\ dA = -\int_{plate}\tau_w\ dx$ $-\vec{D} = \rho\int_{(1)} U(-U)\ dA + \rho\int_{(2)} u^2 dA \quad \vec{D} = \rho U^2 bh - \rho b\int_0^\delta u^2\ dy$ $Uh = \int_0^\delta u\ dy \quad \rho U^2 bh = \rho b\int_0^\delta Uu\ dy \quad \vec{D} = \rho b\int_0^\delta u(U-u)dy$ $\vec{D} = \rho bU^2\Theta \quad \frac{d\vec{D}}{dx} = \rho bU^2\frac{d\Theta}{dx} \quad \frac{d\vec{D}}{dx} = b\tau_w \quad \tau_w = \rho U^2\frac{d\Theta}{dx}$ $\vec{D} = \rho b\int_0^\delta u(U-u)dy = \rho bU^2\delta\int_0^1 g(Y)[1-g(Y)]dY$ $\vec{D} = \rho bU^2\delta C_1 \quad C_1 = \int_0^1 g(Y)[1-g(Y)]dY$ $\tau_w = \mu\frac{\partial u}{\partial y}\Big\rvert_{y=0} = \frac{\mu U}{\delta}\frac{dg}{dY}\Big\rvert_{Y=0} = \frac{\mu U}{\delta}C_2 \quad C_2 = \frac{dg}{dY}\Big\rvert_{Y=0}$ $\delta\ d\delta = \frac{\mu C_2}{\rho UC_1}dx \quad \delta = \sqrt{\frac{2\nu C_2 x}{UC_1}} \quad \frac{\delta}{x} = \frac{\sqrt{2C_2/C_1}}{\sqrt{\mathrm{Re}_x}}$ $\tau_w = \sqrt{\frac{C_1C_2}{2}}U^{3/2}\sqrt{\frac{\rho\mu}{x}} \quad c_f = \frac{\tau_w}{\frac{1}{2}\rho U^2}$ $c_f = \sqrt{2C_1C_2}\sqrt{\frac{\mu}{\rho Ux}} = \frac{\sqrt{2C_1C_2}}{\sqrt{\mathrm{Re}_x}} \quad c_f = \frac{0.664}{\sqrt{\mathrm{Re}_x}}$	[four operations] (M1N3) → 1st (2A) [exponent] (M6A3) → 6th (2A) [dot product] → To be taught as a special math topic [square root] (M8N1) → 8th (2A) [sigma notation] (M6N1) → 6th (1A) or (MA1A3) → 9th (2E) [integration] → 12th (To be taught as a special skill) [partial derivative] → Post-secondary	[velocity] (S8P3)→ 8th (3A) [density] (S6E5) → 6th (4A) [gravity] (S6E1) → 6th (3A)	PS	9th + PS

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code)** **[Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 9 Flow over Immersed Bodies (Continued)				
9.2.3 Momentum Integral boundary Layer Equation for a Flat Plate (Continued) $C_{Df}=\frac{\vec{D}_f}{\frac{1}{2}\rho U^2 b\ell}=\frac{b\int_0^\ell \tau_w dx}{\frac{1}{2}\rho U^2 b\ell}$ $C_{Df}=\frac{1}{\ell}\int_0^\ell c_f dx$ $C_{Df}=\frac{\sqrt{8C_1C_2}}{\sqrt{\mathrm{Re}_\ell}}$ $C_{Df}=\frac{1.328}{\sqrt{\mathrm{Re}_\ell}}$	[four operations] (M1N3) → 1st (2A) [exponent] (M6A3) → 6th (2A) [dot product] → To be taught as a special math topic [square root] (M8N1) → 8th (2A) [sigma notation] (M6N1) → 6th (1A) or (MA1A3) → 9th (2E) [integration] → 12th (To be taught as a special skill) [partial derivative] → Post-secondary	[velocity] (S8P3)→ 8th (3A) [density] (S6E5) → 6th (4A) [gravity] (S6E1) → 6th (3A)	PS	9th + PS
9.2.4 Transition from Laminar to Turbulent Flow N/A				
9.2.5 Turbulent Boundary Layer Flow N/A				
9.2.6 Effects of Pressure Gradient N/A				
9.2.7 Momentum Integral Boundary Layer Equation with Nonzero Pressure Gradient $\frac{dp}{dx}=-\rho U_{fs}\frac{dU_{fs}}{dx}$ $\tau_w=\rho\frac{d}{dx}\left(U_{fs}^2\Theta\right)+\rho\delta^* U_{fs}\frac{dU_{fs}}{dx}$ $U_{fs}=U=\text{constant}$	[four operations] (M1N3) → 1st (2A) [exponent] (M6A3) → 6th (2A) [partial derivative] → Post-secondary	[velocity] (S8P3)→ 8th (3A) [density] (S6E5) → 6th (4A)	**PS**	
9.3 Drag $C_D=\frac{\vec{D}}{\frac{1}{2}\rho U^2 A}$ $C_D=\phi(shape,\mathrm{Re},Ma,Fr,\varepsilon/\ell)$	[four operations] (M1N3) → 1st (2A) [areas of geometric shapes] (M5M1) → 5th (2B)	[force] (S4P3) → 4th (3A) or (S8P3) → 8th (3C) [density] (S6E5) → 6th (4A)	9th	
9.3.1 Friction Drag $\vec{D}_f=\frac{1}{2}\rho U^2 b\ell C_{Df}$	[four operations] (M1N3) → 1st (2A) [areas of geometric shapes] (M5M1) → 5th (2B)	[force] (S4P3) → 4th (3A) or (S8P3) → 8th (3C) [density] (S6E5) → 6th (4A)	9th	

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 9 Flow over Immersed Bodies (Continued)				
9.3.2 Pressure Drag $D_p = \int \rho \cos\theta \, dA$ $C_{Dp} = \frac{\vec{D}_p}{\frac{1}{2}\rho U^2 A} = \frac{\int \rho \cos\theta \, dA}{\frac{1}{2}\rho U^2 A} = \frac{\int C_p \cos\theta \, dA}{A}$ $\vec{D} = f(U, \ell, \mu)$ $\vec{D} = C\mu\ell U$ $C_D = \frac{\vec{D}}{\frac{1}{2}\rho U^2 \ell^2} = \frac{2C\mu\ell U}{\rho U^2 \ell^2} = \frac{2C}{\mathrm{Re}}$	[four operations] (M1N3) → 1st (2A) [areas of geometric shapes] (M5M1) → 5th (2B) [integration] → 12th (To be taught as a special skill)	[force] (S4P3) → 4th (3A) or (S8P3) → 8th (3C) [density] (S6E5) → 6th (4A)	9th	9th + PS
9.3.3 Drag Coefficient Data and Examples				
9.4 Lift **9.4.1 Surface Pressure Distribution** $C_L = \frac{\vec{L}}{\frac{1}{2}\rho U^2 A}$ $C_L = \phi(shape, \mathrm{Re}, Ma, Fr, \varepsilon/\ell)$	[four operations] (M1N3) → 1st (2A) [areas of geometric shapes] (M5M1) → 5th (2B)	[force] (S4P3) → 4th (3A) or (S8P3) → 8th (3C) [density] (S6E5) → 6th (4A)	9th	
9.4.2 Circulation N/A				
9.5 Chapter Summary and Study Guide N/A	N/A	N/A	9th	
Chapter 10 Open Channel Flow				
10.1 General Characteristics of Open-Channel Flow $\mathrm{Re} = \rho V R_h / \mu$ $Fr = V/(g\ell)^{1/2}$	[four operations] (M1N3) → 1st (2A) [exponent] (M6A3) → 6th (2A) [trigonometric functions] (MA2G2) → 10th (2F) [ellipse] (MA2G4) → 10th (2F) → To be taught	[velocity] (S8P3) → 8th (3A) [gravity] (S6E1) → 6th (3A)	9th	9th + PS

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 10 Open Channel Flow (Continued)				
10.2 Surface Waves **10.2.1 Wave Speed** $\left.\begin{array}{l} -cyb = (-c+\delta V)(y+\delta y)b \\ c = \frac{(y+\delta y)\delta V}{\delta y} \\ \delta y << y \;\rightarrow\; c = y\frac{\delta V}{\delta y} \end{array}\right\} \rightarrow$ $\frac{1}{2}\gamma y^2 b - \frac{1}{2}\gamma(y+\delta y)^2 b = \rho bcy[(c-\delta V)-c]$ $F_1 = \frac{\gamma y_{c1}A_1 = \gamma(y+\delta y)^2 b}{2} \quad F_2 = \frac{\gamma y_{c2}A_2 = \gamma(y+\delta y)^2 b}{2}$ $\frac{\delta V}{\delta y} = \frac{g}{c} \quad c = \sqrt{gy} \quad \left.\begin{array}{l} \frac{V^2}{2g} + y = \text{constant} \\ \frac{V\,\delta V}{g} + \delta y = 0 \\ y\;\delta V + V\;\delta y = 0 \\ \frac{\delta y}{y} << 1 \;\rightarrow\; c \approx \sqrt{gy}\left(1+\frac{\delta y}{y}\right)^{1/2} \end{array}\right\} \rightarrow$	[four operations] (M1N3) → 1st (2A) [square root] (M8N1) → 8th (2A) [trigonometric functions] (MA2G2) → 10th (2F) [derivative] → 12th (To be taught)	[velocity] (S8P3)→ 8th (3A) [speed] (S2P3) → 2nd (3A) [gravity] (S6E1) → 6th (3A)	9th	9th + PS

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 10 Open Channel Flow (Continued)				
10.2.1 Wave Speed (Continued) $c=\left[\frac{g\lambda}{2\pi}\tanh\left(\frac{2\pi y}{\lambda}\right)\right]^{1/2} \quad y>>\lambda \quad \rightarrow \quad c=\sqrt{\frac{g\lambda}{2\pi}}$ $\tanh\left(\frac{2\pi y}{\lambda}\right)\rightarrow 1 \quad as \quad \frac{y}{\lambda}\rightarrow\infty$ $\tanh\left(\frac{2\pi y}{\lambda}\right)\rightarrow\frac{2\pi y}{\lambda} \quad as \quad \frac{y}{\lambda}\rightarrow 0$	[four operations] (M1N3) → 1st (2A) [square root] (M8N1) → 8th (2A) [trigonometric functions] (MA2G2) → 10th (2F) [analytic geometry: hyperbolic tangent] Post-secondary → To be taught [derivative] → 12th (To be taught)	[velocity] (S8P3)→ 8th (3A) [speed] (S2P3) → 2nd (3A) [gravity] (S6E1) → 6th (3A)	9th	9th + PS
10.2.2 Froude Number Effects N/A	N/A	[velocity] (S8P3)→ 8th (3A) [speed] (S2P3) → 2nd (3A)	9th	
10.3 Energy Considerations $\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+h_L \quad \left.\begin{array}{l} z_1-z_2=S_0\ell \\ \frac{p_1}{\gamma}=y_1 \\ \frac{p_2}{\gamma}=y_2 \end{array}\right\} \rightarrow$ $y_1+\frac{V_1^2}{2g}+S_0\ell=y_2+\frac{V_2^2}{2g}+h_L$ $S_f=\frac{h_L}{\ell} \quad \rightarrow \quad y_1-y_2=\frac{\left(V_2^2-V_1^2\right)}{2g}+\left(S_f-S_0\right)\ell$ $\left.\begin{array}{l} S_f=0 \\ S_0=0 \end{array}\right\} \rightarrow \quad y_1-y_2=\frac{\left(V_2^2-V_1^2\right)}{2g}$	[four operations] (M1N3) → 1st (2A) [exponent] (M6A3) → 6th (2A)	[velocity] (S8P3)→ 8th (3A) [gravity] (S6E1) → 6th (3A) [potential energy] (SP3) → 9th (3A)	9th	

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code)** **[Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 10 Open Channel Flow (Continued)				
10.3.1 Specific Energy $E = y + \frac{V^2}{2g}$ $E_1 = E_2 + (S_f - S_0)\ell$ $E = y + \frac{q^2}{2gy^2}$ $\frac{dE}{dy} = 1 - \frac{q^2}{gy^3} = 0$ $y_c = \left(\frac{q^2}{g}\right)^{1/3}$ $E_{min} = \frac{3y_c}{2}$ $V_c = \frac{q}{y_c} = \frac{(y_c^{3/2} g^{1/2})}{y_c} = \sqrt{gy_c}$ $Fr \equiv V_c/(gy_c)^{1/2} = 1$	[four operations] (M1N3) → 1st (2A) [derivative] → 12th (To be taught)	[energy] (SP3) → 9th (3B) [gravity] (S6E1) → 6th (3A) [velocity] (S8P3)→ 8th (3A)	9th	9th + PS
10.3.2 Channel Depth Variations $H_1 = H_2 + h_L$ $\frac{dH}{dx} = S_f$ $\frac{dz}{dx} = S_0$ $\frac{dH}{dx} = \frac{d}{dx}\left(\frac{V^2}{2g} + y + z\right) = \frac{V}{g}\frac{dV}{dx} + \frac{dy}{dx} + \frac{dz}{dx}$ $\frac{dh_L}{dx} = \frac{V}{g}\frac{dV}{dx} + \frac{dy}{dx} + S_0$ $\frac{V}{g}\frac{dV}{dx} + \frac{dy}{dx} = S_f - S_0$ $\frac{dV}{dx} = -\frac{q}{y^2}\frac{dy}{dx} = -\frac{V}{y}\frac{dy}{dx}$ $\frac{V}{g}\frac{dV}{dx} = \frac{V^2}{gy}\frac{dy}{dx} = -Fr^2\frac{dy}{dx}$ $Fr = V/(gy)^{1/2}$ $\frac{dy}{dx} = \frac{(S_f - S_0)}{(1 - Fr^2)}$	[four operations] (M1N3) → 1st (2A) [derivative] → 12th (To be taught)	[velocity] (S8P3)→ 8th (3A) [gravity] (S6E1) → 6th (3A)	PS	
10.4 Uniform Depth Channel Flow **10.4.1 Uniform Flow Approximations** N/A	[areas of geometric shapes] (M5M1) → 5th (2B) [perimeter] (M3M3) (M3M4) → 3rd (2B)	[velocity] (S8P3) → 8th (3A) [stress] → To be taught	9th	

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 10 Open Channel Flow (Continued)				
10.4.2 The Chezy and Manning Equations $\sum F_x = \rho Q(V_2 - V_1) = 0 \quad F_1 - F_2 - \tau_w P\ell + \vec{W}\sin\theta = 0 \quad \sum F_x = 0$ $\left.\begin{matrix} \tau_w = \dfrac{\vec{W}\sin\theta}{P\ell} = \dfrac{\vec{W}S_0}{P\ell} \\ \sin\theta \approx \tan\theta = S_0 \\ S_0 << 1 \\ \vec{W} = \gamma A\ell \\ R_h = A/P \end{matrix}\right\} \rightarrow \begin{matrix} \tau_w = \dfrac{\gamma A\ell S_0}{P\ell} = \gamma R_h S_0 \\ \tau_w == K\rho\dfrac{V^2}{2} \quad K\rho\dfrac{V^2}{2} = \gamma R_h S_0 \quad \rightarrow \\ V = C\sqrt{R_h S_0} \\ V = \dfrac{R_h^{2/3}S_0^{1/2}}{n} \end{matrix}$ $V = \dfrac{\kappa}{n}AR_h^{2/3}S_0^{1/2} \quad Q = \dfrac{\kappa}{n}AR_h^{2/3}S_0^{1/2}$	[four operations] (M1N3) → 1st (2A) [exponent] (M6A3) → 6th (2A) [areas of geometric shapes] (M5M1) → 5th (2B) [trigonometric functions] (MA2G2) → 10th (2F)	[pressure] (SC5) → 9th (4B) → To be taught [velocity] (S8P3)→ 8th (3A)	**9th**	9th + PS
10.4.3 Uniform Depth Examples N/A				
10.5 Gradually Varied Flow N/A				
10.5.1 Classification of Surface Shapes N/A				
10.5.2 Examples of Gradually Varied Flows N/A				
10.6 Rapidly Varied Flow N/A	[four operations] (M1N3) → 1st (2A) [exponent] (M6A3) → 6th (2A) [square root] (M8N1) → 8th (2A)	[force] (S4P3) → 4th (3A) or (S8P3) → 8th (3C) [velocity] (S8P3)→ 8th (3A) [gravity] (S6E1) → 6th (3A) [pressure] (SC5) → 9th (4B) → To be taught	9th	

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 10 Open Channel Flow (Continued)				
10.6.1 The Hydraulic Jump $F_1 - F_2 = \rho Q(V_2 - V_1) = \rho V_1 y_1 b(V_2 - V_1)$ $\left.\begin{array}{l} F_1 = p_{c1}A_1 = \frac{\gamma y_1^2 b}{2} \quad p_{c1} = \frac{\gamma y_1}{2} \\ F_2 = p_{c2}A_2 = \frac{\gamma y_2^2 b}{2} \quad p_{c1} = \frac{\gamma y_2}{2} \end{array}\right\} \rightarrow \frac{y_1^2}{2} - \frac{y_2^2}{2} = \frac{V_1 y_1}{g}(V_2 - V_1)$ $y_1 b_1 V_1 = y_2 b_2 V_2 = Q \quad y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + h_L$ $\frac{y_1^2}{2} - \frac{y_2^2}{2} = \frac{V_1 y_1}{g}\left(\frac{V_1 y_1}{y_2} - V_1\right) = \frac{V_1^2 y_1}{g y_2}(y_1 - y_2)$ $\left(\frac{y_2}{y_1}\right)^2 + \left(\frac{y_2}{y_1}\right) - 2Fr_1^2 = 0 \quad \left.\begin{array}{l} Fr_1 = \frac{V_1}{\sqrt{g y_1}} \\ \frac{y_2}{y_1} = \frac{1}{2}\left(-1 \pm \sqrt{1 + 8Fr_1^2}\right) \end{array}\right\} \rightarrow$ $\frac{y_2}{y_1} = \frac{1}{2}\left(-1 + \sqrt{1 + 8Fr_1^2}\right) \quad \frac{h_L}{y_1} = 1 - \frac{y_2}{y_1} + \frac{Fr_1^2}{2}\left[1 - \left(\frac{y_1}{y_2}\right)^2\right]$	[four operations] (M1N3) → 1st (2A) [exponent] (M6A3) → 6th (2A) [square root] (M8N1) → 8th (2A)	[force] (S4P3) → 4th (3A) or (S8P3) → 8th (3C) [velocity] (S8P3)→ 8th (3A) [gravity] (S6E1) → 6th (3A) [pressure] (SC5) → 9th (4B) → To be taught	9th	9th + PS

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code)** **[Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 10 Open Channel Flow (Continued)				
10.6.2 Sharp-Crested Weirs $\frac{p_A}{\gamma}+\frac{V_1^2}{2g}+z_A=(H+P_w-h)+\frac{u_2^2}{2g}\quad u_2=\sqrt{2g\left(h+\frac{V_1^2}{2g}\right)}$ $Q=\int_{(2)}u_2\,dA=\int_{h=0}^{h=H}u_2\ell\,dh$ $\ell=b\rightarrow Q=\sqrt{2g}b\int_0^H\left(h+\frac{V_1^2}{2g}\right)^{1/2}dh$ $Q=\frac{2}{3}\sqrt{2g}b\left[\left(H+\frac{V_1^2}{2g}\right)^{3/2}-\left(\frac{V_1^2}{2g}\right)^{3/2}\right]\left.\begin{matrix}P_w\gg H\\ \frac{V_1^2}{2g}\ll H\end{matrix}\right\}\rightarrow$ $Q=\frac{2}{3}\sqrt{2g}H^{3/2}\quad Q=C_{wr}\frac{2}{3}\sqrt{2g}bH^{3/2}\quad C_{wr}=0.611+0.075\left(\frac{H}{P_w}\right)$ $\ell=2(H-h)\tan\left(\frac{\theta}{2}\right)\quad \frac{V_1^2}{2g}\ll H\rightarrow Q=\frac{8}{15}\tan\left(\frac{\theta}{2}\right)\sqrt{2g}H^{5/2}$ $Q=C_{wt}\frac{8}{15}\tan\left(\frac{\theta}{2}\right)\sqrt{2g}H^{5/2}$	[four operations] (M1N3) → 1st (2A) [exponent] (M6A3) → 6th (2A) [square root] (M8N1) → 8th (2A) [trigonometric functions] (MA2G2) → 10th (2F) [integration] → 12th (To be taught as a special skill) Nore: The main formulas are not based on calculus.	[velocity] (S8P3)→ 8th (3A) [gravity] (S6E1) → 6th (3A)	9th	9th + PS
10.6.3 Broad-Crested Weirs $H+P_w+\frac{V_1^2}{2g}=y_c+p_w+\frac{V_c^2}{2g}\quad H-y_c=\frac{\left(V_c^2-V_1^2\right)}{2g}=\frac{V_c^2}{2g}$ $\left.\begin{matrix}V_2=V_c=(gy_c)^{1/2}\\ V_c^2=gy_c\end{matrix}\right\}\rightarrow H-y_c=\frac{y_c}{2}\rightarrow y_c=\frac{2H}{3}$	[four operations] (M1N3) → 1st (2A) [exponent] (M6A3) → 6th (2A) [square root] (M8N1) → 8th (2A)	[velocity] (S8P3)→ 8th (3A) [gravity] (S6E1) → 6th (3A)	9th	

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code)** **[Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 10 Open Channel Flow (Continued)				
10.6.3 Broad-Crested Weirs (Continued) $Q = by_2V_2 = by_cV_c = by_c(gy_c)^{1/2} = b\sqrt{g}\,y_c^{3/2} \rightarrow$ $Q = b\sqrt{g}\left(\frac{2}{3}\right)^{3/2} H^{3/2}$ $Q = C_{wb} b\sqrt{g}\left(\frac{2}{3}\right)^{3/2} H^{3/2}$ $C_{wb} = \frac{0.65}{(1+H/P_w)^{1/2}}$	[four operations] (M1N3) → 1st (2A) [exponent] (M6A3) → 6th (2A) [square root] (M8N1) → 8th (2A)	[velocity] (S8P3)→ 8th (3A) [gravity] (S6E1) → 6th (3A)	9th	9th + PS
10.6.4 Underflow Gates $q = C_d a\sqrt{2gy_1}$	[four operations] (M1N3) → 1st (2A) [exponent] (M6A3) → 6th (2A) [square root] (M8N1) → 8th (2A)	[velocity] (S8P3)→ 8th (3A) [gravity] (S6E1) → 6th (3A)	9th	
10.7 Chapter Summary and Study Guide N/A	N/A	N/A	9th	
Chapter 11 Compressible Flow				
11.1 Ideal Gas Relationships $p = \rho RT$ $R = \frac{\lambda}{M_{gas}}$ $c_v = \left(\frac{\partial \check{u}}{\partial T}\right)_v = \frac{d\check{u}}{dT}$ $d\check{u} = c_v dT$ $\check{u}_2 - \check{u}_1 = \int_{T_1}^{T_2} c_v\, dT$ $\not{V} = \frac{1}{\rho}$ $\check{u}_2 - \check{u}_1 = c_v(T_2 - T_1)$ $\check{h} = \check{u} + \frac{p}{\rho}$ $\check{u} = \check{u}(T)$ $\frac{p}{\rho} = RT$ $\check{h} = \check{h}(T)$ $c_p = \left(\frac{\partial \check{h}}{\partial T}\right)_p = \frac{d\check{h}}{dT}$ $d\check{h} = c_p dT$ $\check{h}_2 - \check{h}_1 = \int_{T_1}^{T_2} c_p\, dT$ $\check{h}_2 - \check{h}_1 = c_p(T_2 - T_1)$ $\check{h} = \check{u} + RT$	[four operations] (M1N3) → 1st (2A) [volume] (M5M4) → 5th (1B) (M6M3) → 6th (2B) (MA1G5) → 9th (2F) [functions] (MA1A1) → 9th (2E) and others → Post-secondary [integration] → 12th (To be taught as a special skill) [partial derivative] → Post-secondary	[**Ideal Gas Law**] → Post-secondary → to be taught [heat] (S2P2) → 2nd (3A) [temperature] (SP3) → 9th (3B) [density] (S6E5) → 6th (4A) [pressure] (SC5) → 9th (4B) → To be taught	PS	9th + PS

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code)** **[Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 11 Compressible Flow (Continued)				
11.1 Ideal Gas Relationships (Continued) $d\check{h} = d\check{u} + R\ dT \quad \frac{d\check{h}}{dT} = \frac{d\check{u}}{dT} + R \quad c_p - c_v = R \quad k = \frac{c_p}{c_v}$ $c_p = \frac{Rk}{k-1} \quad c_v = \frac{R}{k-1} \quad T\ ds = d\check{u} + pd\left(\frac{1}{\rho}\right)$ $d\check{h} = d\check{u} + pd\left(\frac{1}{\rho}\right) + \left(\frac{1}{\rho}\right)dp \quad T\ ds = d\check{h} - \left(\frac{1}{\rho}\right)dp$ $ds = c_v \frac{dT}{T} + \frac{R}{1/\rho} d\left(\frac{1}{\rho}\right) \quad ds = c_p \frac{dT}{T} - R\frac{dp}{p}$ $s_2 - s_1 = c_v \ln\frac{T_2}{T_1} + R\ln\frac{\rho_1}{\rho_2} \quad s_2 - s_1 = c_p \ln\frac{T_2}{T_1} + R\ln\frac{p_2}{p_1}$	[four operations] (M1N3) → 1st (2A) [volume] (M5M4) → 5th (1B) (M6M3) → 6th (2B) (MA1G5) → 9th (2F) [functions] (MA1A1) → 9th (2E) and others → Post-secondary [integration] → 12th (To be taught as a special skill) [partial derivative] → Post-secondary	[**Ideal Gas Law**] → Post-secondary → to be taught [heat] (S2P2) → 2nd (3A) [temperature] (SP3) → 9th (3B) [density] (S6E5) → 6th (4A) [pressure] (SC5) → 9th (4B) → To be taught	PS	9th + PS
11.2 Mach Number and Speed of Sound $Ma = \frac{V}{c} \quad \rho Ac = (\rho + \delta\rho)(c - \delta V)$ $\rho c = \rho c - \rho\delta V + c\delta\rho - (\delta\rho)(\delta V) \quad \rho\ \delta V = c\ \delta\rho$ $-c\rho cA + (c - \delta V)(\rho + \delta\rho)(c - \delta V)A = pA - (p + \delta p)A$ $-c\rho cA + (c - \delta V)\rho Ac = -\delta pA \quad \rho\delta V = \frac{\delta p}{c} \quad c^2 = \frac{\delta p}{\delta\rho} \quad c = \sqrt{\frac{\delta p}{\delta\rho}}$ $\frac{\delta p}{\rho} + \delta\left(\frac{V^2}{2}\right) + g\ \delta z = \delta(\text{loss}) \quad \frac{\delta p}{\rho} + \frac{(c - \delta V)^2}{2} - \frac{c^2}{2} = 0$ $\rho\ \delta V = \frac{\delta p}{c}$	[four operations] (M1N3) → 1st (2A) [areas of geometric shapes] (M5M1) → 5th (2B) [exponent] (M6A3) → 6th (2A) [square root] (M8N1) → 8th (2A) [partial derivative] → Post-secondary	[speed of sound] (SPS9) → 9th (3B) [pressure] (SC5) → 9th (4B) → To be taught [density] (S6E5) → 6th (4A) [velocity] (S8P3)→ 8th (3A)	PS	

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code)** **[Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 11 Compressible Flow (Continued)				
11.2 Mach Number and Speed of Sound (Continued) $\left.\begin{matrix} c=\sqrt{\frac{\delta p}{\delta \rho}} \\ \delta p \to \partial p \to 0 \end{matrix}\right\} \to \quad c=\sqrt{\left(\frac{\partial p}{\partial \rho}\right)_s} \quad p=(\text{constant})(\rho^k)$ $\left(\frac{\partial p}{\partial \rho}\right)_s=(\text{constant})k\rho^{k-1}=\frac{p}{\rho^k}k\rho^{k-1}=\frac{p}{\rho}k=RTk \quad c=\sqrt{RTk}$ $E_v=\frac{dp}{d\rho/\rho}=\rho\left(\frac{\partial p}{\partial \rho}\right)_s \quad c=\sqrt{\frac{E_v}{\rho}}$	[four operations] (M1N3) → 1^{st} (2A) [volume] (M5M4) → 5^{th} (1B) (M6M3) → 6^{th} (2B) (MA1G5) → 9^{th} (2F) [functions] (MA1A1) → 9^{th} (2E) and others → Post-secondary [integration] → 12^{th} (To be taught as a special skill) [partial derivative] → Post-secondary	[**Ideal Gas Law**] → Post-secondary → to be taught [heat] (S2P2) → 2^{nd} (3A) [temperature] (SP3) → 9^{th} (3B) [density] (S6E5) → 6^{th} (4A) [pressure] (SC5) → 9^{th} (4B) → To be taught	PS	9^{th} + PS
11.3 Categories of Compressible Flow $r=(t-t_{wave})c \quad \sin\alpha=\frac{c}{V}=\frac{1}{Ma}$	[four operations] (M1N3) → 1^{st} (2A) [trigonometric functions] (MA2G2) → 10^{th} (2F)	[velocity] (S8P3)→ 8^{th} (3A) [speed of sound] (SPS9) → 9^{th} (3B)	9^{th}	
11.4 Isentropic Flow of an Ideal Gas **11.4.1 Effect of Variations in Flow Cross-Sectional Areas** $\dot{m}=\rho AV=\text{constant} \quad dp+\frac{1}{2}\rho\, d(V^2)+\gamma\, dz=0 \quad \frac{dp}{\rho V^2}=-\frac{dV}{V}$ $\ln\rho+\ln A+\ln V=\text{constant} \quad \frac{d\rho}{\rho}+\frac{dA}{A}+\frac{dV}{V}=0 \quad \to$ $\left.\begin{matrix} -\frac{dV}{V}=\frac{d\rho}{\rho}+\frac{dA}{A} \\ \frac{dp}{\rho V^2}=-\frac{dV}{V} \end{matrix}\right\} \to \quad \frac{dp}{\rho V^2}\left(1-\frac{V^2}{dp/d\rho}\right)=\frac{dA}{A}$	[four operations] (M1N3) → 1^{st} (2A) [areas of geometric shapes] (M5M1) → 5^{th} (2B) [exponent] (M6A3) → 6^{th} (2A) [partial derivatives] → Post-secondary	[pressure] (SC5) → 9^{th} (4B) → To be taught [velocity] (S8P3) → 8^{th} (3A) [density] (S6E5) → 6^{th} (4A)	PS	

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code)** **[Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 11 Compressible Flow (Continued)				
11.4.1 Effect of Variations in Flow Cross-Sectional Areas (Continued) $\left.\begin{array}{l} c=\sqrt{\left(\frac{\partial p}{\partial \rho}\right)_s} \\ Ma=\frac{V}{c} \\ \frac{dp}{\rho V^2}\left(1-\frac{V^2}{dp/d\rho}\right)=\frac{dA}{A}\end{array}\right\} \rightarrow \frac{dp}{\rho V^2}\left(1-Ma^2\right)=\frac{dA}{A}$ $\left.\begin{array}{l} \frac{dp}{\rho V^2}=-\frac{dV}{V} \\ \frac{dp}{\rho V^2}\left(1-Ma^2\right)=\frac{dA}{A}\end{array}\right\} \rightarrow \frac{dV}{V}=-\frac{dA}{A}\frac{1}{\left(1-Ma^2\right)}$ $\frac{d\rho}{\rho}=\frac{dA}{A}\frac{Ma^2}{\left(1-Ma^2\right)}$ $\frac{dA}{dV}=-\frac{A}{V}\left(1-Ma^2\right)$	[four operations] (M1N3) → 1st (2A) [areas of geometric shapes] (M5M1) → 5th (2B) [exponent] (M6A3) → 6th (2A) [partial derivatives] → Post-secondary	[pressure] (SC5) → 9th (4B) → To be taught [velocity] (S8P3) → 8th (3A) [density] (S6E5) → 6th (4A)	PS	9th + PS
11.4.2 Converging-Diverging Duct Flow $\frac{p}{p^k}=\text{constant}=\frac{p_0}{p_0^k}$ $\frac{dp}{\rho}+d\left(\frac{V^2}{2}\right)=0$ $\frac{p_0^{1/k}}{\rho_0}\frac{dp}{(p)^{1/k}}+d\left(\frac{V^2}{2}\right)=0$ $\frac{k}{k-1}\left(\frac{p_0}{\rho_0}-\frac{p}{\rho}\right)-\frac{V^2}{2}=0$ $\frac{kR}{k-1}\left(T_0-T\right)-\frac{V^2}{2}=0$ $\left.\begin{array}{l} c_p\left(T_0-T\right)-\frac{V^2}{2}=0 \\ \check{h}_2-\check{h}_1=c_p\left(T_2-T_1\right)\end{array}\right\} \rightarrow \check{h}_0-\left(\check{h}+\frac{V^2}{2}\right)=0$	[four operations] (M1N3) → 1st (2A) [exponent] (M6A3) → 6th (2A) [square root] (M8N1) → 8th (2A)	[pressure] (SC5) → 9th (4B) → To be taught [density] (S6E5) → 6th (4A) [velocity] (S8P3) → 8th (3A) [**Ideal Gas Law**] → Post-secondary → to be taught	9th	

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 11 Compressible Flow (Continued)				
11.4.2 Converging-Diverging Duct Flow (Continued) $\frac{T}{T_0}=\frac{1}{1+[(k-1)/2]Ma^2}$ $\left(\frac{p}{p_0}\right)\left(\frac{\rho_0}{\rho}\right)=\frac{T}{T_0}$ $\left(\frac{p}{p_0}\right)=\left(\frac{T}{T_0}\right)^{k/(k-1)}$ $\left.\begin{matrix}\frac{T}{T_0}=\frac{1}{1+[(k-1)/2]Ma^2}\\ \left(\frac{p}{p_0}\right)=\left(\frac{T}{T_0}\right)^{k/(k-1)}\end{matrix}\right\}\rightarrow \frac{p}{p_0}=\left\{\frac{1}{1+[(k-1)/2]Ma^2}\right\}^{k/(k-1)}$ $\left.\begin{matrix}\frac{T}{T_0}=\frac{1}{1+[(k-1)/2]Ma^2}\\ \left(\frac{p}{p_0}\right)\left(\frac{\rho_0}{\rho}\right)=\frac{T}{T_0}\\ \frac{p}{p_0}=\left\{\frac{1}{1+[(k-1)/2]Ma^2}\right\}^{k/(k-1)}\end{matrix}\right\}\rightarrow$ $\frac{\rho_0}{\rho}=\left\{\frac{1}{1+[(k-1)/2]Ma^2}\right\}^{k/(k-1)}$ $\frac{p^*}{p_0}=\left(\frac{2}{k+1}\right)^{k/(k-1)}$ $\left(\frac{p^*}{p_0}\right)_{k=1.4}=0.528$ $p^*_{k=1.4}=0.528p_{atm}$ $\frac{T^*}{T_0}=\frac{2}{k+1}$ $\left(\frac{T^*}{T_0}\right)_{k=1.4}=0.833$ $T^*_{k=1.4}=0.833T_{atm}$	[four operations] (M1N3) → 1st (2A) [exponent] (M6A3) → 6th (2A) [square root] (M8N1) → 8th (2A)	[pressure] (SC5) → 9th (4B) → To be taught [density] (S6E5) → 6th (4A) [velocity] (S8P3) → 8th (3A) [**Ideal Gas Law**] → Post-secondary → to be taught	9th	9th + PS

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code)** **[Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 11 Compressible Flow (Continued)				
11.4.2 Converging-Diverging Duct Flow (Continued) $\left.\begin{array}{l} Ma = 1 \\ p = \rho RT \\ \frac{p^*}{p_0} = \left(\frac{2}{k+1}\right)^{k/(k-1)} \\ \frac{T^*}{T_0} = \frac{2}{k+1} \end{array}\right\} \rightarrow$ $\frac{\rho^*}{\rho_0} = \left(\frac{\rho^*}{T^*}\right)\left(\frac{T_0}{p_0}\right) = \left(\frac{2}{k+1}\right)^{k/(k-1)}\left(\frac{k+1}{2}\right) = \left(\frac{2}{k+1}\right)^{k/(k-1)}$ $\left(\frac{\rho^*}{\rho_0}\right)_{k=1.4} = 0.634$ $\rho AV = \rho^* A^* V^*$ $\frac{A}{A^*} = \left(\frac{\rho^*}{\rho}\right)\left(\frac{V^*}{V}\right)$ $V^* = \sqrt{RT^* k}$, $V = Ma\sqrt{RTk}$ $\frac{A}{A^*} = \frac{1}{Ma}\left(\frac{\rho^*}{\rho_0}\right)\left(\frac{\rho_0}{\rho}\right)\sqrt{\frac{(T^*/T_0)}{T/T_0}}$	[four operations] (M1N3) → 1st (2A) [exponent] (M6A3) → 6th (2A) [square root] (M8N1) → 8th (2A)	[pressure] (SC5) → 9th (4B) → To be taught [density] (S6E5) → 6th (4A) [velocity] (S8P3) → 8th (3A) [**Ideal Gas Law**] → Post-secondary → to be taught	9th	9th + PS

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code)** **[Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 11 Compressible Flow (Continued)				
11.4.2 Converging-Diverging Duct Flow (Continued) $\frac{T}{T_0}=\frac{1}{1+[(k-1)/2]Ma^2}$ $\frac{p}{p_0}=\left\{\frac{1}{1+[(k-1)/2]Ma^2}\right\}^{k/(k-1)}$ $\frac{T*}{T_0}=\frac{2}{k+1}$ $\frac{\rho*}{\rho_0}=\left(\frac{\rho*}{T*}\right)\left(\frac{T_0}{p_0}\right)=\left(\frac{2}{k+1}\right)^{k/(k-1)}\left(\frac{k+1}{2}\right)=\left(\frac{2}{k+1}\right)^{k/(k-1)}$ $\frac{A}{A*}=\frac{1}{Ma}\left(\frac{\rho*}{\rho_0}\right)\left(\frac{\rho_0}{\rho}\right)\sqrt{\frac{(T*/T_0)}{T/T_0}}$ } → $\frac{A}{A*}=\frac{1}{Ma}\left\{\frac{1+[(k-1)/2]Ma^2}{1+[(k-1)/2]}\right\}^{(k+1)/[2(k-1)]}$	[four operations] (M1N3) → 1st (2A) [exponent] (M6A3) → 6th (2A) [square root] (M8N1) → 8th (2A)	[pressure] (SC5) → 9th (4B) → To be taught [density] (S6E5) → 6th (4A) [velocity] (S8P3) → 8th (3A) [**Ideal Gas Law**] → Post-secondary → to be taught	**9th**	9th + PS
11.4.3 Constant Area Duct Flow N/A	N/A	[density] (S6E5) → 6th (4A) [velocity] (S8P3)→ 8th (3A) [pressure] (SC5) → 9th (4B) [friction] (S8P3) → 8th (3A) → To be taught [acceleration] (S8P3) → 8th (3C)	**9th**	

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code)** **[Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 11 Compressible Flow (Continued)				
11.5 Nonisentropic Flow of an Ideal Gas **11.5.1 Adiabatic Constant Area Duct Flow with Friction (Fanno Flow)** $\dot{m}\left[\check{h}_2-\check{h}_1+\frac{V_2^2-V_1^2}{2}+g(z_2-z_1)\right]=\dot{Q}_{\substack{net\\in}}+\dot{W}_{\substack{shaft\\net\ in}}$ $\left.\begin{matrix}\check{h}+\frac{V^2}{2}=\check{h}_0=\text{constant}\\ \check{h}-\check{h}_0=c_p(T-T_0)\end{matrix}\right\}\rightarrow\left\{\begin{matrix}T+\frac{V^2}{2c_p}=T_0=\text{constant}\\ T+\frac{(\rho V)^2}{2c_p\rho^2}=T_0=\text{constant}\end{matrix}\right.$ $T+\frac{(\rho V)^2T^2}{2c_p(p^2/R^2)}=T_0=\text{constant}\quad\leftarrow\quad p=\rho RT\quad\uparrow$ $s-s_1=c_p\ln\frac{T}{T_1}-R\ln\frac{p}{p_1}$ $T\,ds=d\check{h}-\frac{dp}{\rho}\quad\left.\begin{matrix}d\check{h}=c_pdT\\ p=\rho RT\end{matrix}\right\}\rightarrow\frac{dp}{p}=\frac{d\rho}{\rho}+\frac{dT}{T}$ $T\,ds=c_pdT-RT\left(\frac{d\rho}{\rho}+\frac{dT}{T}\right)\quad\rho V=\text{constant}\quad\frac{d\rho}{\rho}=-\frac{dV}{V}\rightarrow$ $T\,ds=c_pdT-RT\left(-\frac{dV}{V}+\frac{dT}{T}\right)\quad\frac{ds}{dT}=\frac{c_p}{T}-R\left(-\frac{1}{V}\frac{dV}{dT}+\frac{1}{T}\right)$ $\frac{dV}{dT}=-\frac{c_p}{V}\quad\frac{ds}{dT}=\frac{c_p}{T}-R\left(\frac{c_p}{V^2}+\frac{1}{T}\right)\quad V_a=\sqrt{RT_ak}$	[four operations] (M1N3) → 1st (2A) [areas of geometric shapes] (M5M1) → 5th (2B) [exponent] (M6A3) → 6th (2A) [logarithmic functions] (MA2A5) → 10th (2E) (To be taught as a special skill) [square root] (M8N1) → 8th (2A) [integration] → 12th (To be taught) [derivative] → Post-secondary	[velocity] (S8P3)→ 8th (3A) [density] (S6E5) → 6th (4A) [**Ideal Gas Law**] → Post-secondary → to be taught [temperature] (SP3) → 9th (3B) [entropy] → Post-secondary → To be taught [pressure] (SC5) → 9th (4B) → To be taught [momentum] (SP3) → 9th (3B) [pressure] (SC5) → 9th (4B) [friction] (S8P3) → 8th (3A) → To be taught [wave] (S8P4) → 8th (3A)	PS	9th + PS

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code)** **[Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 11 Compressible Flow (Continued)				
11.5.1 Adiabatic Constant Area Duct Flow with Friction (Fanno Flow) $p_1A_1 - p_2A_2 - R_x = \dot{m}(V_2 - V_1)$ $\quad p_1 - p_2 - \frac{R_x}{A} = \rho V(V_2 - V_1)$ $-dp - \frac{\tau_w \pi D\, d_x}{A} = \rho V\, dV \quad f = \frac{8\tau_w}{\rho V^2}$ $\uparrow \ A = \frac{\pi D^2}{4} \ \rightarrow \ -dp - f\rho\frac{V^2}{2}\frac{dx}{D} = \rho V\, dV$ $dp + \frac{f}{p}\frac{\rho V^2}{2}\frac{dx}{D} + \frac{\rho}{p}\frac{d(V^2)}{2} = 0$ $\frac{dp}{p} + \frac{fk}{2}Ma^2\frac{dx}{D} + k\frac{Ma^2}{2}\frac{d(V^2)}{V^2} = 0$ $V^2 = Ma^2RTk \quad \frac{d(V^2)}{V^2} = \frac{d(Ma^2)}{Ma^2} + \frac{dT}{T} \quad \frac{dT}{T} + \frac{d(V^2)}{2c_pT} = 0$ $\frac{dT}{T} + \frac{k-1}{2}Ma^2\frac{d(V^2)}{V^2} = 0 \quad \frac{d(V^2)}{V^2} = \frac{d(Ma^2)/Ma^2}{1+[(k-1)/2]Ma^2}$ $\frac{dp}{p} = \frac{1}{2}\frac{d(V^2)}{V^2} - \frac{d(Ma^2)}{Ma^2}$ $\frac{1}{2}(1+kMa^2)\frac{d(V^2)}{V^2} - \frac{d(Ma^2)}{Ma^2} + \frac{f}{k}Ma^2\frac{dx}{D} = 0$	[four operations] (M1N3) → 1st (2A) [areas of geometric shapes] (M5M1) → 5th (2B) [exponent] (M6A3) → 6th (2A) [logarithmic functions] (MA2A5) → 10th (2E) (To be taught as a special skill) [square root] (M8N1) → 8th (2A) [integration] → 12th (To be taught) [derivative] → Post-secondary	[velocity] (S8P3)→ 8th (3A) [density] (S6E5) → 6th (4A) [**Ideal Gas Law**] → Post-secondary → to be taught [temperature] (SP3) → 9th (3B) [entropy] → Post-secondary → To be taught [pressure] (SC5) → 9th (4B) → To be taught [momentum] (SP3) → 9th (3B) [pressure] (SC5) → 9th (4B) [friction] (S8P3) → 8th (3A) → To be taught [wave] (S8P4) → 8th (3A)	PS	9th + PS

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code)** **[Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 11 Compressible Flow (Continued)				
11.5.1 Adiabatic Constant Area Duct Flow with Friction (Fanno Flow) (Continued) $\frac{(1-Ma^2)d(Ma^2)}{\{1+[(k-1)/2]Ma^2\}kMa^4} = f\frac{dx}{D}$ $\int_{Ma}^{Ma^*=1} \frac{(1-Ma^2)d(Ma^2)}{\{1+[(k-1)/2]Ma^2\}kMa^4} = \int_{\ell}^{\ell^*} f\frac{dx}{D}$ $\frac{1}{k}\frac{(1-Ma^2)}{Ma^2} + \frac{k+1}{2k}\ln\left\{\frac{[(k+1)/2]Ma^2}{1+[(k-1)/2]Ma^2}\right\} = \frac{f(\ell^*-\ell)}{D}$ $\frac{f(\ell^*-\ell_2)}{D} - \frac{f(\ell^*-\ell_1)}{D} = \frac{f}{D}(\ell_1-\ell_2)$ $\frac{dT}{T} = -\frac{(k-1)}{2\{1+[(k-1)/2]Ma^2\}}d(Ma^2)$ $\frac{T}{T^*} = \frac{(k+1)/2}{1+[(k-1)/2]Ma^2} \quad \frac{V}{V^*} = \frac{Ma\sqrt{RTk}}{\sqrt{RT^*k}} = Ma\sqrt{\frac{T}{T^*}}$ $\frac{V}{V^*} = \left\{\frac{[(k+1)/2]Ma^2}{1+[(k-1)/2]Ma^2}\right\}^{1/2} \quad \frac{\rho}{\rho^*} = \frac{V^*}{V}$ $\frac{\rho}{\rho^*} = \left\{\frac{1+[(k-1)/2]Ma^2}{[(k+1)/2]Ma^2}\right\}^{1/2} \quad \frac{p}{p^*} = \frac{\rho}{\rho^*}\frac{T}{T^*}$ $\frac{p}{p^*} = \frac{1}{Ma}\left\{\frac{(k+1)/2}{1+[(k-1)/2]Ma^2}\right\}^{1/2} \quad \frac{p_0}{p^*_0} = \left(\frac{p_0}{p}\right)\left(\frac{p}{p^*}\right)\left(\frac{p^*}{p^*_0}\right)$ $\frac{p_0}{p^*_0} = \frac{1}{Ma}\left[\left(\frac{2}{k+1}\right)\left(1+\frac{k-1}{2}Ma^2\right)\right]^{(k+1)/2(k-1)}$	[four operations] (M1N3) → 1^{st} (2A) [areas of geometric shapes] (M5M1) → 5^{th} (2B) [exponent] (M6A3) → 6^{th} (2A) [logarithmic functions] (MA2A5) → 10^{th} (2E) (To be taught as a special skill) [square root] (M8N1) → 8^{th} (2A) [integration] → 12^{th} (To be taught) [derivative] → Post-secondary	[velocity] (S8P3)→ 8^{th} (3A) [density] (S6E5) → 6^{th} (4A) [**Ideal Gas Law**] → Post-secondary → to be taught [temperature] (SP3) → 9^{th} (3B) [entropy] → Post-secondary → To be taught [pressure] (SC5) → 9^{th} (4B) → To be taught [momentum] (SP3) → 9^{th} (3B) [pressure] (SC5) → 9^{th} (4B) [friction] (S8P3) → 8^{th} (3A) → To be taught [wave] (S8P4) → 8^{th} (3A)	PS	9^{th} + PS

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code)** **[Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 11 Compressible Flow (Continued)				
11.5.2 Frictionless Constant Area Duct Flow with Heat Transfer (Rayleigh Flow) $p_1 A_1 + \dot{m} V_1 = p_2 A_2 + \dot{m} V_2 + R_x \quad p + \frac{(\rho V)^2}{\rho} = \text{constant}$ $p + \frac{(\rho V)^2 RT}{\rho} = \text{constant} \quad \rho \text{V} = \text{constant}$ $dp = -\rho V\, dV \quad \frac{dp}{\rho} = -V\, dV \quad T\, ds = d\,\check{h} + V\, dV$ $T\, ds = c_p\, dT + V\, dV \quad \frac{ds}{dT} = \frac{c_p}{T} + \frac{V}{T}\frac{dV}{dT}$ $\frac{ds}{dT} = \frac{c_p}{T} + \frac{V}{T}\frac{1}{[(T/V) - (V/R)]} \quad V_a = \sqrt{RT_a k} \quad Ma_a = 1$ $\frac{dT}{ds} = \frac{1}{ds/dT} = \frac{1}{(c_p/T) + (V/T)[(T/V) - (V/R)]^{-1}}$ $\frac{dT}{ds} = 0 \quad \rightarrow \quad Ma_b = \sqrt{\frac{1}{k}}$ $d\,\check{h} + V\, dV = \delta q \quad \frac{dV}{V} = \frac{\delta q}{c_p T}\left[\frac{V}{T}\frac{dT}{dV} + \frac{V^2(k-1)}{kRT}\right]^{-1}$ $p + \rho V^2 = p_a + \rho_a V_a^2$ $\frac{dV}{V} = \frac{\delta q}{c_p T}\frac{1}{(1 - Ma^2)} \quad \frac{p}{p_a} + \frac{\rho V^2}{p_a} = 1 + \frac{\rho_a}{p_a} V_a^2$ $\frac{p}{p_a} = \frac{1+k}{1+kMa^2} \quad \frac{T}{T_a} = \frac{p}{p_a}\frac{\rho_a}{\rho} \quad \frac{\rho_a}{\rho} = \frac{V}{V_a}$	[four operations] (M1N3) → 1st (2A) [exponent] (M6A3) → 6th (2A) [derivative] → 12th + [square root] (M8N1) → 8th (2A)	[velocity] (S8P3)→ 8th (3A) [density] (S6E5) → 6th (4A) [temperature] (SP3) → 9th (3B) [pressure] (SC5) → 9th (4B) → To be taught	PS	9th + PS

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code)** **[Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 11 Compressible Flow (Continued)				
11.5.2 Frictionless Constant Area Duct Flow with Heat Transfer (Rayleigh Flow) (Continued) $\frac{\rho_a}{\rho} = Ma\sqrt{\frac{T}{T_a}}$ $\frac{T}{T_a} = \left(\frac{p}{p_a}Ma\right)^2$ $\frac{T}{T_a} = \left[\frac{(1+k)Ma}{1+kMa^2}\right]^2$ $\frac{\rho_a}{\rho} = \frac{V}{V_a} = Ma\left[\frac{(1+k)Ma}{1+kMa^2}\right]$ $\frac{T}{T_{0,a}} = \left(\frac{T_0}{T}\right)\left(\frac{T}{T_a}\right)\left(\frac{T_a}{T_{0,a}}\right)$ $\frac{T}{T_{0,a}} = \frac{2(k+1)Ma^2\left(1+\frac{k-1}{2}Ma^2\right)}{(1+kMa^2)^2}$ $\frac{p_0}{p_{0,a}} = \left(\frac{p_0}{p}\right)\left(\frac{p}{p_a}\right)\left(\frac{p_a}{p_{0,a}}\right)$ $\frac{p_0}{p_{0,a}} = \frac{(1+k)}{(1+kMa^2)}\left[\left(\frac{2}{k+1}\right)\left(1+\frac{k-1}{2}\right)Ma^2\right]^{k/(k-1)}$	[four operations] (M1N3) → 1^{st} (2A) [exponent] (M6A3) → 6^{th} (2A) [derivative] → 12^{th} + [square root] (M8N1) → 8^{th} (2A)	[velocity] (S8P3)→ 8^{th} (3A) [density] (S6E5) → 6^{th} (4A) [temperature] (SP3) → 9^{th} (3B) [pressure] (SC5) → 9^{th} (4B) → To be taught	PS	9^{th} + PS
11.5.3 Normal Shock Waves $\rho V = \text{constant}$ $p+\rho V^2 = \text{constant}$ $p + \frac{(\rho V)^2 RT}{p} = \text{constant}$ $\check{h} + \frac{V^2}{2} = \check{h}_0 = \text{constant}$ $\check{h} - \check{h}_0 = c_p(T-T_0)$ $p = \rho RT$ $T + \frac{(\rho V)^2 T^2}{2c_p(p^2/R^2)} = T_0 = \text{constant}$ $\frac{p_y}{p_x} = \left(\frac{p_y}{p_a}\right)\left(\frac{p_a}{p_x}\right)$ $\frac{p_y}{p_a} = \frac{1+k}{1+kMa_y^2}$ $\frac{p_x}{p_a} = \frac{1+k}{1+kMa_x^2}$	[four operations] (M1N3) → 1^{st} (2A) [exponent] (M6A3) → 6^{th} (2A)	[**Ideal Gas Law**] → Post-secondary → to be taught [temperature] (S3P1) → 3^{rd} (3A) [density] (S6E5) → 6^{th} (4A) [pressure] (SC5) → 9^{th} (4B) → To be taught [speed] (S2P3) → 2^{nd} (3A) [velocity] (S8P3)→ 8^{th} (3A) [graph] (S7CS6) → 7^{th} (6)	9^{th}	

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code)** **[Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 11 Compressible Flow (Continued)				
11.5.3 Normal Shock Waves (Continued) $\frac{p_y}{p_x} = \frac{1+k\text{Ma}_x^2}{1+k\text{Ma}_y^2}$ $\frac{p_y}{p_x} = \left(\frac{p_y}{p^*}\right)\left(\frac{p^*}{p_x}\right)$ $\frac{p}{p^*} = \frac{1}{\text{Ma}}\left\{\frac{(k+1)/2}{1+[(K-1)/2]\text{Ma}^2}\right\}^{1/2}$ $p_x + \rho_x V_x^2 = p_y + \rho_y V_Y^2$ $\frac{\rho V^2}{p} = \frac{V^2}{RT} = \frac{kV^2}{RTk} = k\text{Ma}^2$ $\frac{T_y}{T_x} = \left(\frac{T_y}{T^*}\right)\left(\frac{T^*}{T_x}\right) \left. \begin{matrix} \frac{T_y}{T^*} = \frac{(k+1)/2}{1+[(k-1)/2]\text{Ma}_y^2} \\ \frac{T_x}{T^*} = \frac{(k+1)/2}{1+[(k-1)/2]\text{Ma}_x^2} \end{matrix} \right\} \rightarrow$ $\frac{T_y}{T_x} = \frac{1+[(k-1)/2]\text{Ma}_x^2}{1+[(k-1)/2]\text{Ma}_y^2}$ $\frac{p_y}{p_x} = \left(\frac{T_y}{T_x}\right)\left(\frac{\rho_y}{\rho_x}\right)$ $\rho_x V_x = \rho_y V_y$ $\frac{p_y}{p_x} = \left(\frac{T_y}{T_x}\right)\left(\frac{V_x}{V_y}\right)$ $\frac{p_y}{p_x} = \left(\frac{T_y}{T_x}\right)^{1/2}\left(\frac{\text{Ma}_x}{\text{Ma}_y}\right)$ $\frac{p_y}{p_x} = \left\{\frac{1+[(k-1)/2]\text{Ma}_x^2}{1+[(k-1)/2]\text{Ma}_y^2}\right\}^{1/2} \frac{\text{Ma}_x}{\text{Ma}_y}$ $\text{Ma}_y^2 = \frac{\text{Ma}_x^2 + [2/(k-1)]}{[2k/(k-1)]\text{Ma}_x^2 - 1}$ $\frac{p_y}{p_x} = \frac{2k}{k+1}\text{Ma}_x^2 - \frac{k-1}{k+1}$ $\frac{T_y}{T_x} = \frac{\{1+[(k-1)/2]\text{Ma}_x^2\}\{2k/(k-1)\}\text{Ma}_x^2 - 1}{\{(k+1)^2/2(k-1)\}\text{Ma}_x^2}$	[four operations] (M1N3) → 1st (2A) [exponent] (M6A3) → 6th (2A)	[**Ideal Gas Law**] → Post-secondary → to be taught [temperature] (S3P1) → 3rd (3A) [density] (S6E5) → 6th (4A) [pressure] (SC5) → 9th (4B) → To be taught [speed] (S2P3) → 2nd (3A) [velocity] (S8P3)→ 8th (3A) [graph] (S7CS6) → 7th (6)	9th	9th + PS

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 11 Compressible Flow (Continued)				
11.5.3 Normal Shock Waves (Continued) $\frac{\rho_y}{\rho_x}=\frac{V_x}{V_y}$ $\frac{\rho_y}{\rho_x}=\left(\frac{p_y}{p_x}\right)\left(\frac{T_x}{T_y}\right)$ $\frac{\rho_y}{\rho_x}=\frac{V_x}{V_y}=\frac{(k+1)\mathrm{Ma}_x^2}{(k-1)\mathrm{Ma}_x^2+2}$ $\frac{p_{0,y}}{p_{0,x}}=\left(\frac{p_{0,y}}{p_y}\right)\left(\frac{p_y}{p_x}\right)\left(\frac{p_x}{p_{0,x}}\right)$ $\frac{p_{0,y}}{p_{0,x}}=\frac{\left(\frac{k+1}{2}\mathrm{Ma}_x^2\right)^{k/(k-1)}\left(1+\frac{k-1}{2}\mathrm{Ma}_x^2\right)^{k/(1-k)}}{\left(\frac{2k}{k+1}\mathrm{Ma}_x^2-\frac{k-1}{k+1}\right)^{1/(k-1)}}$	[four operations] (M1N3) → 1st (2A) [exponent] (M6A3) → 6th (2A)	[**Ideal Gas Law**] → Post-secondary → to be taught [temperature] (S3P1) → 3rd (3A) [density] (S6E5) → 6th (4A) [pressure] (SC5) → 9th (4B) → To be taught [speed] (S2P3) → 2nd (3A) [velocity] (S8P3)→ 8th (3A) [graph] (S7CS6) → 7th (6)	9th	9th + PS
11.6 Analogy between Compressible and Open-Channel Flows $Ma=\frac{V}{c}$ $Fr=\frac{V_{oc}}{\sqrt{gy}}$ $c_{oc}=\sqrt{gy}$ $Fr=\frac{V_{oc}}{c_{oc}}$ $\rho AV=\text{constant}$ $ybV_{oc}=\text{constant}$ $\mathrm{c}=\sqrt{(\text{constant})k\rho^{k-1}}$	[four operations] (M1N3) → 1st (2A) [square root] (M8N1) → 8th (2A) [areas of geometric shapes] (M5M1) → 5th (2B)	[density] (S6E5) → 6th (4A) [velocity] (S8P3)→ 8th (3A) [gravity] (S6E1) → 6th (3A) [mass] (S8P3) → 8th (3A)	9th	
11.7 Two-Dimensional Compressible Flow $V_{t1}=V_{t2}$	[four operations] (M1N3) → 1st (2A) [triangle] (M5M1) → 5th (2B)	[velocity] (S8P3)→ 8th (3A)	9th	
11.8 Chapter Summary and Study Guide N/A	N/A	N/A	9th	
Chapter 12 Turbomachines				
12.1 Introduction N/A	N/A	[force] (S4P3) → 4th (3A) or (S8P3) → 8th (3C) [work] (S8P3) → 8th (3A) [energy] (SP3) → 9th (3B) [power] (SP3) → 9th (3B)	9th	9th

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 12 Turbomachines (Continued)				
12.2 Basic Energy Considerations $\vec{V} = \vec{W} + \vec{U} \quad U = \omega r$	[four operations] (M1N3) → 1st (2A) [radius] (M3G1) → 3rd (2B)	[velocity] (S8P3)→ 8th (3A)	9th	9th
12.3 Basic Angular Momentum Considerations $\sum\left(\vec{r} \times \vec{F}\right) = \int_{cs} \left(\vec{r} \times \vec{V}\right) \rho \vec{V} \cdot \hat{n}\, dA \quad T_{shaft} = -\dot{m}_1 \left(r_1 V_{\theta 1}\right) + -\dot{m}_2 \left(r_1 V_{\theta 2}\right)$ $m = \rho Q \quad \dot{W}_{shaft} = T_{shaft} \omega \quad \dot{W}_{shaft} = -\dot{m}_1 \left(U_1 V_{\theta 1}\right) + -\dot{m}_2 \left(U_1 V_{\theta 2}\right)$ $w_{shaft} = \frac{\dot{W}_{shaft}}{\dot{m}} \quad w_{shaft} = -U_1 V_{\theta 1} + U_1 V_{\theta 2} \quad V^2 = V_\theta^2 + V_x^2$ $V_x^2 + \left(V_\theta - U\right)^2 = W^2 \quad V_\theta U = \frac{V^2 + U^2 - W^2}{2}$ $w_{shaft} = \frac{V_2^2 - V_1^2 + U_2^2 - U_1^2 - \left(W_2^2 - W_1^2\right)}{2}$	[sigma notation] (M6N1) → 6th (1A) or (MA1A3) → 9th (2E) [integration] → 12th (To be taught as a special skill) [special math: cross product] → To be taught as a special math topic [analytic geometry] → 12th (To be taught) [areas of geometric shapes] (M5M1) → 5th (2B)	[density] (S6E5) → 6th (4A) [torque] → Post-secondary → To be taught [momentum] (SP3) → 9th (3B)	9th	
12.4 The Centrifugal Pump N/A	N/A	N/A	9th	
12.4.1 Theoretical Considerations $\vec{V}_1 = \vec{W}_1 + \vec{U}_1 \quad U_1 = r_1 \omega$ $\vec{V}_2 = \vec{W}_2 + \vec{U}_2 \quad U_2 = r_2 \omega \quad \dot{m}_1 = \dot{m}_2 = \dot{m}$ $T_{shaft} = \dot{m}\left(r_2 V_{\theta 2} - r_1 V_{\theta 1}\right) \quad T_{shaft} = \rho Q \left(r_2 V_{\theta 2} - r_1 V_{\theta 1}\right)$ $\dot{W}_{shaft} = T_{shaft} \omega \quad \dot{W}_{shaft} = \rho Q \omega \left(r_2 V_{\theta 2} - r_1 V_{\theta 1}\right)$ $\dot{W}_{shaft} = \rho Q \left(U_2 V_{\theta 2} - U_1 V_{\theta 1}\right) \quad w_{shaft} = \frac{\dot{W}_{shaft}}{\rho Q} = U_2 V_{\theta 2} - U_1 V_{\theta 1}$ $\dot{W}_{shaft} = \rho g Q h_i \quad h_i = \frac{1}{g}\left(U_2 V_{\theta 2} - U_1 V_{\theta 1}\right)$	[four operations] (M1N3) → 1st (2A) [triangle] (M5M1) → 5th (2B) [trigonometric functions] (MA2G2) → 10th (2F) [areas of geometric shapes] (M5M1) → 5th (2B)	[velocity] (S8P3)→ 8th (3A) [density] (S6E5) → 6th (4A) [gravity] (S6E1) → 6th (3A)	9th	

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code)** **[Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 12 Turbomachines (Continued)				
12.4.1 Theoretical Considerations (Continued) $h_i = \frac{1}{2g}\left[\left(V_2^2 - V_1^2\right) + \left(U_2^2 - U_1^2\right) + \left(W_2^2 - W_1^2\right)\right]$ $h_i = \frac{U_2 V_{\theta 2}}{g}$ $\cot\beta_2 = \frac{U_2 - V_{\theta 2}}{V_{r2}}$ $h_i = \frac{U_2^2}{g} - \frac{U_2 V_{r2}\cot\beta_2}{g}$ $Q = 2\pi r_2 b_2 V_{r2}$ $h_i = \frac{U_2^2}{g} - \frac{U_2 \cot\beta_2}{2\pi r_2 b_2 g} Q$	[four operations] (M1N3) → 1st (2A) [triangle] (M5M1) → 5th (2B) [trigonometric functions] (MA2G2) → 10th (2F) [areas of geometric shapes] (M5M1) → 5th (2B)	[velocity] (S8P3)→ 8th (3A) [density] (S6E5) → 6th (4A) [gravity] (S6E1) → 6th (3A)	9th	9th
12.4.2 Pump Performance Characteristics $h_a = \frac{p_2 - p_1}{\gamma} + z_2 - z_1 + \frac{V_2^2 - V_1^2}{2g}$ $h_a = h_p = h_s - h_L$ $h_a \approx \frac{p_2 - p_1}{\gamma}$ $\wp_f = \gamma Q h_a$ $\wp_f = \text{water horsepower} = \frac{\gamma Q h_a}{550}$ $\eta = \frac{\text{power gained by thevfluid}}{\text{shaft power driving the pump}} = \frac{\wp_f}{\dot{W}_{shaft}}$ $\eta = \frac{\gamma Q h_a / 550}{bhp}$ $\eta = \eta_h \eta_m \eta_v$	[four operations] (M1N3) → 1st (2A) [areas of geometric shapes] (M5M1) → 5th (2B) [unit conversion] (M6M1) → 6th (2C)	[pressure] (SC5) → 9th (4B) → To be taught [velocity] (S8P3)→ 8th (3A) [gravity] (S6E1) → 6th (3A)	9th	
12.4.3 Net Positive Suction Head (NPSH) $NPSH = \frac{p_s}{\gamma} + \frac{V_s^2}{2g} - \frac{p_v}{\gamma}$ $\frac{p_{atm}}{\gamma} - z_1 = \frac{p_s}{\gamma} + \frac{V_s^2}{2g} = \sum h_L \rightarrow$ $\frac{p_s}{\gamma} + \frac{V_s^2}{2g} = \frac{p_{atm}}{\gamma} - z_1 - \sum h_L \rightarrow$ $NPSH = \frac{p_{atm}}{\gamma} - z_1 - \sum h_L - \frac{p_v}{\gamma}$	[four operations] (M1N3) → 1st (2A) [sigma notation] (M6N1) → 6th (1A) or (MA1A3) → 9th (2E)	[pressure] (SC5) → 9th (4B) → To be taught [velocity] (S8P3)→ 8th (3A) [gravity] (S6E1) → 6th (3A) [density] (S6E5) → 6th (4A)	9th	

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code)** **[Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 12 Turbomachines (Continued)				
12.4 4 System Characteristics and Pump Selection $h_p = z_2 - z_1 + \sum h_L \quad h_p = z_2 - z_1 + KQ^2$	[four operations] (M1N3) → 1^{st} (2A) [sigma notation] (M6N1) → 6^{th} (1A) or (MA1A3) → 9^{th} (2E)	[velocity] (S8P3)→ 8^{th} (3A) [density] (S6E5) → 6^{th} (4A)	9^{th}	9^{th}
12.5 Dimensionless Parameters and Similarity Laws $\text{dependent variable} = f(D, \ell_i, \varepsilon, Q, \omega, \mu, \rho)$ $\text{dependent pi term} = \phi\left(\frac{\ell_i}{D}, \frac{\varepsilon}{D}, \frac{Q}{\omega D^3}, \frac{\rho\omega D^2}{\mu}\right)$ $C_H = \frac{gh_a}{\omega^2 D^2} = \phi_1\left(\frac{\ell_i}{D}, \frac{\varepsilon}{D}, \frac{Q}{\omega D^3}, \frac{\rho\omega D^2}{\mu}\right)$ $C_\wp = \frac{\dot{W}_{shaft}}{\rho\omega^3 D^5} = \phi_2\left(\frac{\ell_i}{D}, \frac{\varepsilon}{D}, \frac{Q}{\omega D^3}, \frac{\rho\omega D^2}{\mu}\right)$ $\eta = \frac{\rho g Q h_a}{\dot{W}_{shaft}} = \phi_3\left(\frac{\ell_i}{D}, \frac{\varepsilon}{D}, \frac{Q}{\omega D^3}, \frac{\rho\omega D^2}{\mu}\right)$ $\frac{gh_a}{\omega^2 D^2} = \phi_1\left(\frac{Q}{\omega D^3}\right) \quad \frac{\dot{W}_{shaft}}{\rho\omega^3 D^5} = \phi_2\left(\frac{Q}{\omega D^3}\right) \quad \eta = \phi_3\left(\frac{Q}{\omega D^3}\right)$ $\left(\frac{Q}{\omega D^3}\right)_1 = \left(\frac{Q}{\omega D^3}\right)_2 \quad \left(\frac{gh_a}{\omega^2 D^2}\right)_1 = \left(\frac{gh_a}{\omega^2 D^2}\right)_2$ $\left(\frac{\dot{W}_{shaft}}{\rho\omega^3 D^5}\right)_1 = \left(\frac{\dot{W}_{shaft}}{\rho\omega^3 D^5}\right)_2 \quad \eta = \eta_2$	[four operations] (M1N3) → 1^{st} (2A) [ratio] (M6A1) → 6th (2A)	[gravity] (S6E1) → 6^{th} (3A) [density] (S6E5) → 6^{th} (4A) [energy] (SP3) → 9^{th} (3B) [velocity] (S8P3)→ 8^{th} (3A)	9^{th}	
12.5.1 Special Pump Scaling Laws $\frac{Q_1}{Q_2} = \frac{\omega_1}{\omega_2} \quad \frac{h_{a1}}{h_{a2}} = \frac{\omega_1^2}{\omega_2^2} \quad \frac{\dot{W}_{shaft1}}{\dot{W}_{shaft2}} = \frac{\omega_1^3}{\omega_2^3} \quad \frac{Q_1}{Q_2} = \frac{D_1^3}{D_2^3} \quad \frac{h_{a1}}{h_{a2}} = \frac{D_1^2}{D_2^2}$ $\frac{\dot{W}_{shaft1}}{\dot{W}_{shaft2}} = \frac{D_1^5}{D_2^5} \quad \frac{1-\eta_2}{1-\eta_1} \approx \left(\frac{D_1}{D_2}\right)^{1/5}$	[four operations] (M1N3) → 1^{st} (2A) [areas of geometric shapes: circle, triangle] (M5M1) → 5^{th} (2B) (M5M1) → 5^{th} (2B) [exponent] (M6A3) → 6^{th} (2A) [ratio] (M6A1) → 6th (2A)	[velocity] (S8P3)→ 8^{th} (3A) [power] (SP3) → 9^{th} (3B) [energy] (SP3) → 9^{th} (3B)	9^{th}	

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code)** **[Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 12 Turbomachines (Continued)				
12.5.2 Specific Speed $\frac{(Q/\omega D^3)^{1/2}}{(gh_a/\omega^2 D^2)^{3/4}} = \frac{\omega\sqrt{Q}}{(gh_a)^{3/4}} = N_s \quad N_{sd} = \frac{\omega(rpm)\sqrt{Q(gpm)}}{[h_a(ft)]^{3/4}}$	[four operations] (M1N3) → 1st (2A) [ratio] (M6A1) → 6th (2A)	[speed] (S2P3) → 2nd (3A)	9th	9th
12.5.3 Suction Specific Speed $S_s = \frac{\omega\sqrt{Q}}{[g(NPSH_R)]^{3/4}} \quad S_{sd} = \frac{\omega(rpm)\sqrt{Q(gpm)}}{[NPSH_R(ft)]^{3/4}}$	[four operations] (M1N3) → 1st (2A) [ratio] (M6A1) → 6th (2A)	[speed] (S2P3) → 2nd (3A)	9th	
12.6 Axial-Flow and Mixed-Flow Pump N/A	[graph] (S7CS6) → 7th (6)	[speed] (S2P3) → 2nd (3A)	9th	
12.7 Fans $\left(\frac{p_a}{\rho\omega^2 D^2}\right)_1 = \left(\frac{p_a}{\rho\omega^2 D^2}\right)_2$	[four operations] (M1N3) → 1st (2A) [areas of geometric shapes: circle, triangle] (M5M1) → 5th (2B) (M5M1) → 5th (2B) [ratio] (M6A1) → 6th (2A)	[speed] (S2P3) → 2nd (3A) [pressure] (SC5) → 9th (4B) → To be taught [density] (S6E5) → 6th (4A)	9th	
12.8 Turbines **12.8.1 Impulse Turbines** $V_{\theta 1} = V_1 = W_1 + U \quad V_{\theta 2} = W_2 \cos\beta + U$ $V_{\theta 2} - V_{\theta 1} = (U - V_1)(1-\cos\beta) \quad T_{shaft} = \dot{m} r_m (U - V_1)(1-\cos\beta)$ $\dot{W}_{shaft} = T_{shaft}\omega = \dot{m} U (U - V_1)(1-\cos\beta) \quad U\big\|_{\substack{\max \\ power}} = \frac{V^2}{2}$	[four operations] (M1N3) → 1st (2A) [trigonometric functions] (MA2G2) → 10th (2F) [derivative] → 12th (To be taught)	[power] (SP3) → 9th (3B) [speed] (S2P3) → 2nd (3A) [force] (S4P3) → 4th (3A) or (S8P3) → 8th (3C)	9th	

Table 2. Fluid Mechanics Topic List (Continued).

Engineering Subject: Fluid				
Engineering Analytic Topics & Typical Formulas	**Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**		**Possible Grade to Start the Topic**	
	Math	**Physics/Chemistry**	**Sec**	**Ch**
Chapter 12 Turbomachines (Continued)				
12.8.2 Reaction Turbines $C_Q = \frac{Q}{\omega D^3}$ $C_H = \frac{gh_T}{\omega^2 D^2}$ $C_\wp = \frac{\dot{W}_{shaft}}{\rho\omega^3 D^5}$ $\eta = \frac{\dot{W}_{shaft}}{\rho g Q h_T}$ $C_H = \phi_1(C_Q)$ $C_\wp = \phi_2(C_Q)$ $\eta = \phi_3(C_Q)$ $\eta = \frac{C_\wp}{C_H C_Q}$ $N'_s = \frac{\omega\sqrt{\dot{W}_{shaft}/\rho}}{(gh_T)^{5/4}}$ $N'_{sd} = \frac{\omega(rpm)\sqrt{\dot{W}_{shaft}(bhp)}}{[h_T(ft)]^{5/4}}$	[four operations] (M1N3) → 1st (2A) [square root] (M8N1) → 8th (2A) [exponent] (M6A3) → 6th (2A)	[power] (SP3) → 9th (3B) [speed] (S2P3) → 2nd (3A) [force] (S4P3) → 4th (3A) or (S8P3) → 8th (3C) [density] (S6E5) → 6th (4A) [gravity] (S6E1) → 6th (3A)	9th	9th
12.9 Compressible Flow Turbomachines **12.9.1 Compressors** $\left(\frac{R\dot{m}\sqrt{kRT_{01}}}{D^2 p_{01}}\right)_{test} = \left(\frac{R\dot{m}\sqrt{kRT_{01}}}{D^2 p_{01}}\right)_{std}$ $\dot{m}_{std} = \frac{\dot{m}_{std}\sqrt{T_{01\ test}/T_{0\ std}}}{p\grave{}_{01\ test}/p_{0\ std}}$ $\frac{ND}{\sqrt{kRT_{01}}}$ $N_{std} = \frac{N}{\sqrt{T_{01}/T_{std}}}$	[four operations] (M1N3) → 1st (2A) [square root] (M8N1) → 8th (2A) [graph] (S7CS6) → 7th (6)	[mass] (S8P3) → 8th (3A) [pressure] (SC5) → 9th (4B) [friction] (S8P3) → 8th (3A) → To be taught [velocity] (S8P3)→ 8th (3A) [temperature] (S3P1) → 3rd (3A)	9th	
12.9.2 Compressible Flow Turbines N/A	N/A	[mass] (S8P3) → 8th (3A) [pressure] (SC5) → 9th (4B) → To be taught [friction] (S8P3) → 8th (3A) → To be taught [velocity] (S8P3)→ 8th (3A) [temperature] (S3P1) → 3rd (3A)	9th	
12.10 Chapter Summary and Study Guide N/A	N/A	N/A	9th	
THE END				

Table 3A. Pre-Calculus Based Fluid Mechanics Topics That Possibly Could Be Taught at 9th Grade

Chapter/Section	Page Numbers	Number of Pages
Chapter 1 – Introduction (pp. 1-30 → 30 pages sub-total. 10 sections out of 11)		
1.1 Some Characteristics of Fluid	1-13	13
1.2 Dimensions, Dimensional Homogeneity, and Units		
1.3 Analysis of Fluid Mechanics Behavior		
1.4 Measures of Fluid Mechanics Mass and Weight		
1.4.1 Density		
1.4 2 Specific Weight		
1.4.3 Specific Gravity		
1.5 Ideal Gas Law		
1.7 Compressibility of Fluids	20-30	11
1.7.1 Bulk Modulus		
1.7.2 Compression and Expansion of Gases		
1.7.3 Speed of Sound		
1.8 Vapor Pressure		
1.9 Surface Tension		
1.10 A Brief Look Back in History		
1.11 Chapter Summary and Study Guide		
Chapter 2 Fluid Statics (pp. 38-79 → 42 pages sub-total. 9 sections out of 13)		
2.3 Pressure Variation in a Fluid at Rest (Concept only)*		
2.3.1 Incompressible Fluid	42-56	15
2.3.2 Compressible Fluid		
2.4 Standard Atmosphere		
2.5 Measurement of Pressure		
2.6 Monometry		
2.6.1 Piezometer Tube		
2.6.2 U-Tube Manometer		
2.6.3 Inclined-Tube Manometer		
2.7 Mechanical and Electronic Pressure Measuring Devices		
2.9 Pressure Prism	63-72	10
2.10 Hydrostatic Force on a Curves Surface		
2.11 Buoyancy, Flotation, and Stability		
2.11.1 Archimedes' Principle		
2.11.2 Stability		
2.13 Chapter Summary and Study Guide	78-79	2

* Basic principles covered under this section heading could be explored; but the formulas used are calculus-based.

Table 3A. Pre-Calculus Based Fluid Mechanics Topics That Possibly Could Be Taught at 9th Grade (Continued)

Chapter/Section	Page Numbers	Number of Pages
Chapter 3 Elementary Fluid Dynamics – The Bernoulli Equation (pp. 95-135 → 41 pages sub-total. 8 sections out of 9)		
3.1 Newton's Second Law	95-101	7
3.2 F = ma along a Streamline		
3.4 Physical Interpretation	104-135	32
3.5 Static, Stagnation, Dynamic, and Total Pressure		
3.6 Examples of Use of the Bernoulli Equation		
3.6.1 Free Jets		
3.6.2 Confined Flows		
3.6.3 Flowrate Measurement		
3.7 The Energy Line and the Hydraulic Grade Line		
3.8 Restrictions on Use of the Bernoulli Equation		
3.8.1 Compressibility Effects		
3.8.3 Rotational Effects		
3.8.4 Other Restrictions		
3.9 Chapter Summary and Study Guide		
Chapter 4 Fluid Kinematics (pp. 150-184 → 35 pages sub-total. 3 sections out of 5)		
4.3 Control Volume and System Representations	168-169	2
4.4 The Reynolds Transport Theorem	170-171	2
4.4.7 Selection of a Control Volume	182-182	3
4.5 Chapter Summary and study Guide		
Chapter 5 Finite Control Volume Analysis (pp. 192-252 → 61 pages sub-total 2 sections out of 5)		
5.1 Conservation of Mass – The Continuity Equation (Concept only)*		
5.1.2 Fixed, Non-deforming Control Volume	195-200	6
5.3.3 Comparison of the Energy Equation with the Bernoulli Equation	236-246	11
5.3.4 Application of the Energy Equation to Non-uniform Flow		
5.3.5 Combination of the Energy Equation and the Moment-of-momentum Equation		
5.4.4 Application of the Loss Form of the Energy Equation	249-252	4
5.5 Chapter Summary and Study Guide		
Chapter 6 Differential Analysis of Fluid Flow (pp. 272-334 → 63 pages sub-total. 0 sections out of 11)		
Chapter 7 Similitude, Dimensional Analysis, and Modeling (pp. 346-391 → 46 pages sub-total. 0 sections out of 11)		

* Basic principles covered under this section heading could be explored; but the formulas used are calculus-based.

Table 3A. Pre-Calculus Based Fluid Mechanics Topics That Possibly Could Be Taught at 9th Grade (Continued)

Chapter/Section	Page Numbers	Number of Pages
Chapter 8 Viscous Flow in Pipes (pp. 401-472 → 72 pages sub-total. 5 sections out of 7)		
8.2 Fully Developed Laminar Flow (Concept only)*		
8.2.4 Energy Considerations	416-417	2
8.4 Dimensional Analysis of Pipe Flow	430-472	43
8.4.1 Major Losses		
8.4.2 Minor Losses		
8.4.3 Noncircular Conduits		
8.5 Pipe Flow Examples		
8.5.1 Single Pipes		
8.5.2 Multiple Pipe Systems		
8.6 Pipe Flowrate Measurement		
8.6.1 Pipe Flowrate Meters		
8.6.2 Volume Flow Meters		
8.7 Chapter Summary and Study Guide		
Chapter 9 Flow over Immersed Bodies (pp. 483-550 → 68 pages sub-total. 4 sections out of 5)		
9.1 General External Flow Characteristics	484-493	10
9.1.1 Lift and Drag Concepts		
9.1.2 Characteristics of Flow Past an Object		
9.3 Drag	518-550	33
9.3.1 Friction Drag		
9.3.2 Pressure Drag		
9.3.3 Drag Coefficient Data and Examples		
9.4 Lift		
9.4.1 Surface Pressure Distribution		
9.4.2 Circulation		
9.5 Chapter Summary and Study Guide		
Chapter 10 Open Channel Flow (Whole Chapter; pp. 561-605 → 45 pages sub-total. 7 sections out of 7)		
10.1 General Characteristics of Open-Channel Flow	561-573	13
10.2 Surface Waves		
10.2.1 Wave Speed		
10.2.2 Froude Number Effects		
10.3 Energy Considerations		
10.3.1 Specific Energy		

* Basic principles covered under this section heading could be explored; but the formulas used are calculus-based.

Table 3A. Pre-Calculus Based Fluid Mechanics Topics That Possibly Could Be Taught at 9th Grade (Continued)

Chapter/Section	Page Numbers	Number of Pages
Chapter 10 Open Channel Flow (Continued)		
10.4 Uniform Depth Channel Flow	574-605	32
10.4.1 Uniform Flow Approximations		
10.4.2 The Chezy and Manning Equations		
10.4.3 Uniform Depth Examples		
10.5 Gradually Varied Flow		
10.5.1 Classification of Surface Shapes		
10.5.2 Examples of Gradually Varied Flows		
10.6 Rapidly Varied Flow		
10.6.1 The Hydraulic Jump		
10.6.2 Sharp-Crested Weirs		
10.6.3 Broad-Crested Weirs		
10.6.4 Underflow Gates		
10.7 Chapter Summary and Study Guide		
Chapter 11 Compressible Flow (pp. 614-678 → 65 pages sub-total. 6 sections out of 8)		
11.3 Categories of Compressible Flow	623-628	6
11.4 Isentropic Flow of an Ideal Gas	631-646	
11.4.2 Converging-Diverging Duct Flow		
11.4.3 Constant Area Duct Flow		
11.5 Non-isentropic Flow of an Ideal Gas	665-678	14
11.5.3 Normal Shock Waves		
11.6 Analogy between Compressible and Open-Channel Flows		
11.7 Two-Dimensional Compressible Flow		
11.8 Chapter Summary and Study Guide		
Chapter 12 Turbomachines (Whole Chapter; pp. 684-736 → 53 pages sub-total. 10 sections out of 10)		
12.1 Introduction	684-736	53
12.2 Basic Energy Considerations		
12.3 Basic Angular Momentum Considerations		
12.4 The Centrifugal Pump		
12.4.1 Theoretical Considerations		
12.4.2 Pump Performance Characteristics		
12.4.3 Net Positive Suction Head (NPSH)		
12.4 4 System Characteristics and Pump Selection		

Table 3A. Pre-Calculus Based Fluid Mechanics Topics That Possibly Could Be Taught at 9th Grade (Continued)

Chapter/Section	Page Numbers	Number of Pages
12.5 Dimensionless Parameters and Similarity Laws	↑	↑
12.5.1 Special Pump Scaling Laws		
12.5.2 Specific Speed		
12.5.3 Suction Specific Speed		
12.6 Axial-Flow and Mixed-Flow Pump		
12.7 Fans		
12.8 Turbines		
12.8.1 Impulse Turbines		
12.8.2 Reaction Turbines		
12.9 Compressible Flow Turbomachines		
12.9.1 Compressors		
12.9.2 Compressible Flow Turbines		
12.10 Chapter Summary and Study Guide		

Statistical Summary

Total Number of Pages Covered by Text (Excluding "Problems" Section)	621
Total Numbers of Sections Covered Under All Chapters	64 out of 102

Percentage of Pre-Calculus Sections

$$\%_{\text{Pre-Calculus}} = \left(\frac{\text{Number of Pre-Calculus Sections}}{\text{Total Number of Sections}}\right)(100\%) = \left(\frac{64}{102}\right)(100\%) = 62.7\%$$

Total Numbers of Chapters Covered	10 out of 12

Percentage of Chapters with Pre-Calculus Sections

$$\%_{\text{Pre-Calculus}} = \left(\frac{\text{Number of Chapters with Pre-Calculus Sections}}{\text{Total Number of Chapters}}\right)(100\%) = \left(\frac{10}{12}\right)(100\%) = 83.3\%$$

Total Number of Pages Covered by Pre-Calculus Portion	317

Percentage of Pre-Calculus Volume

$$\%_{\text{Pre-Calculus}} = \left(\frac{\text{Number of Pre-Calculus Pages}}{\text{Total Number of Pages}}\right)(100\%) = \left(\frac{317}{621}\right)(100\%) = 51.0\%$$

Table 3B. Pre-Requisite Mathematics and Science Topics to Be Reviewed Before Teaching the Pre-Calculus Portion of Fluid Mechanics Topics to 9^{th} Grade Students

Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) **[Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**	
Math	**Physics/Chemistry**
1. [analytic geometry] → 12^{th} (To be taught as a special skill)	1. [**absolute temperature**] (SP3) → 9^{th} (3B) → To be taught
2. [analytic geometry: hyperbolic tangent] Post-secondary → To be taught	2. [acceleration] (S8P3) → 8th (3C)
3. [areas of geometric shapes: circle, triangle, etc.] (M5M1) → 5^{th} (2B)	3. [Dimensional Analysis] → Special topics from 7.1 to be taught
4. [cylinder] (M1G1) (M1G2) → 1^{st} (2B)	4. [density] (S6E5) → 6^{th} (4A)
5. [derivative] → 12^{th} (To be taught as a special skill)	5. [energy] (SP3) → 9^{th} (3B)
6. cross product] → To be taught as a special math topic	6. [force] (S4P3) → 4^{th} (3A) or (S8P3) → 8^{th} (3C)
7. [ellipse] (MA2G4) → 10^{th} (2F) → To be taught	7. [friction] (S8P3) → 8^{th} (3A) → To be taught
8. [exponent] (M6A3) → 6^{th} (2A)	8. **[gas/liquid]** (SPS5) → 9^{th} (3B) → To be taught
9. [four operations] (M1N3) → 1^{st} (2A)	9. [graph] (S7CS6) → 7^{th} (6)
10. [graph] (S7CS6) → 7^{th} (6)	10. [gravity] (S6E1) → 6^{th} (3A)
11. [height] (MKM1) → K (2B)	11. [heat] (S2P2) → 2^{nd} (3A)
12. [integration] → 12^{th} (To be taught as a special skill)	12. [**Ideal Gas Law**] → Post-secondary → to be taught
13. [logarithmic functions] (MA2A5) → 10^{th} (2E) (To be taught as a special skill)	13. **[intermolecular cohesive force]** → To be taught
14. [perimeter] (M3M3) (M3M4) → 3^{rd} (2B)	14. [mass] (S8P3) → 8^{th} (3A)
15. [Pythagorean Theorem] (M8G2) → 8^{th} (2B)	15. [molecule] (S8P1)→ 8^{th} (4A)
16. [prism] (M6G2) → 6^{th} (2B)	16. [momentum] (SP3) → 9^{th} (3B)
17. [radius] (M3G1) → 3^{rd} (2B)	17. [Newton's 1^{st}, 2^{nd} and 3^{rd} Laws] (SP1) → 9^{th} (3C) → To be taught
18. [ratio] (M6A1) → 6th (2A)	18. [potential energy] (SP3) → 9^{th} (3A)
19. [sigma notation] (M6N1) → 6^{th} (1A) or (MA1A3) → 9^{th} (2E)	19. [power] (SP3) → 9^{th} (3B)
20. [square root] (M8N1) → 8^{th} (2A)	20. [pressure] (SC5) → 9^{th} (4B) → To be taught
21. [triangle] (M5M1) → 5^{th} (2B)	21. [Reynolds Number] → To be taught as special topic
22. [trigonometric functions] (MA2G2) → 10^{th} (2F)	22. [speed] (S2P3) → 2^{nd} (3A)
23. [unit conversion] (M6M1) → 6^{th} (2C)	23. [speed of sound] (SPS9) → 9^{th} (3B) → To be taught
24. [volume] (M5M4) → 5^{th} (1B) (M6M3) → 6^{th} (2B) (MA1G5) → 9^{th} (2F)	24. [stress] → To be taught
	25. [temperature] (S3P1) → 3^{rd} (3A) and (SP3) → 9^{th} (3B)
	26. [torque] → Post-secondary → To be taught
	27. [velocity] (S8P3)→ 8^{th} (3A)
	28. [weight] (MKM1) → K (2C)
	29. [work] (S8P3) → 8^{th} (3A)

Table 4A. Calculus Based Fluid Mechanics Topics for Post-Secondary Engineering Education

Chapter/Section	Page Nos.	Chapter/Section	Page Nos.
Chapter 1 – Introduction		**Chapter 4 Fluid Kinematics**	
1.6 Viscosity	13-20	4.1 The Velocity Field	150-168
Chapter 2 Fluid Statics		4.1.1 Eulerian and Lagrangian Flow Descriptions	
2.1 Pressure at a Point	38-42	4.1.2 one-, Two-, and three-Dimensional Flows	
2.2 Basic Equation for Pressure Field		4.1.3 Steady and Unsteady Flows	
2.3 Pressure Variation in a Fluid Mechanics at Rest		4.1.4 Streamilnes, Streaklines, and Pathlines	
2.8 Hydrostatic Force on a Plane Surface	57-63	4.2 The Acceleration Field	
2.12 Pressure Variation in a Fluid Mechanics with Rigid-Body Motion	73-78	4.2.1 The Material Derivative	
2.12.1 Linear Motion		4.2.2 Unsteady Effects	
2.12.2 Rigid-Body Rotation		4.2.3 Convective Effects	
Chapter 3 Elementary Fluid Dynamics – The Bernoulli Equation		4.2.4 Streamline Coordinates	
		4.4 The Reynolds Transport Theorem	170-182
3.2 F = ma along a Streamline (Continued)	97-104	4.4.1 Derivation of the Reynolds Transport Theorem	
3.3 F = ma Normal to a Streamline		4.4.2 Physical Interpretation	
3.8.2 Unsteady Effects	131-132	4.4.3 Relationship to Material Derivative	
Chapter 5 Finite Control Volume Analysis		4.4.4 Steady Effects	
5.1 Conservation of Mass – The Continuity Equation	193-195	4.4.5 Unsteady Effects	
5.1.1 Derivation of the Continuity Equation		4.4.6 Moving Control Volumes	
5.1.3 Moving, Non-deforming Control Volume	200-205	**Chapter 6 Differential Analysis of Fluid Flow**	
5.1.4 Deforming Control Volume		6.1 Fluid Mechanics Element Kinematics	272-334
5.2.1 Derivation of the Linear Momentum Equation	205-236	6.1.1 Velocity and Acceleration Fields Revisited	
5.2.2 Application of the Linear Momentum Equation		6.1.2 Linear Motion and Deformation	
5.2.3 Derivation of the Moment-of-Momentum Equation		6.1.3 Angular Motion and Deformation	
5.2.4 Application of the Moment-of-Momentum Equation		6.2 Conservation of mass	
5.3 First Law of Thermodynamics – The Energy Equation		6.2.1 Differential Survey Form of Continuity Equation	
5.3.1 Derivation of the Energy Equation		6.2.2 Cylindrical Polar Coordinates	
5.3.2 Application of the Energy Equation		6.2.3 The Stream Function	
5.4 Second Law of Thermodynamics – Irreversible Flow	246-249	6.3 Conservation of Linear Momentum	
5.4.1 Semi-infinitesimal Control Volume Statement of the Energy Equation		6.3.1 Description of Forces Acting on the Differential Element	
		6.3.2 Equations of Motion	
5.4.2 Semi-infinitesimal Control Volume Statement of the Second Law of Thermodynamics		6.4 Inviscid Flow	
		6.4.1 Euler's Equations of Motion	
5.4.3 Combination of the Equations of the First and Second Laws of Thermodynamics		6.4.2 The Bernoulli Equation	
		6.4.3 Irrotational Flow	

Table 4A. Calculus Based Fluid Mechanics Topics for Post-Secondary Engineering Education (Continued)

Chapter/Section	Page Nos.	Chapter/Section	Page Nos.
Chapter 7 Similitude, Dimensional Analysis, and Modeling		**Chapter 6 Differential Analysis of Fluid Flow (Continued)**	
7.1 Dimensional Analysis	346-391	6.4.4 The Bernoulli Equation for Irrotational Flow	↑
7.2 Buckingham Pi Theorem		6.4.5The Velocity Potential	
7.3 Determination of Pi Terms		6.5 Some Basic, Plane Potential Flows	
7.4 Some Additional Comments about Dimensional Analysis		6.5.1 Uniform Flow	
7.4.1 Selection of Variables		6.5.2 Source and Sink	
7.4.2 Determination of Reference Dimensions		6.5.3 Vortex	
7.4.3 Uniqueness of Pi Terms		6.5.4 Doublet	
7.5 Determination of Pi Terms by Inspection		6.6 Superposition of Basic, Plane Potential Flows	
7.6 Common Dimensionless Groups in Fluid Mechanics		6.6.1 Source in a Uniform Stream – Half-Body	
7.7 Correlation of Experimental Data		6.6.2 Rankine Ovals	
7.7.1 Problems with One Pi Term		6.6.3 Flow around a Circular Cylinder	
7.7.2 Problems with Two or More Pi Term		6.7 Other Aspects of Potential Flow Analysis	
7.8 Modeling and Similitude		6.8 Viscous Flow	
7.8.1 Theory of Models		6.8.1 Stress-Deformation Relationships	
7.8.2 Model Space		6.8.2 The Navier-Stokes Equations	
7.8.3 Practical Aspects of Using Models		6.9 Some Simple Solutions for Viscous, Incompressible Fluids	
7.9 Some Typical Model Studies		6.9.1 Steady, Laminar Flow between Fixed Parallel Plates	
7.9.1 Flow through Closed Conduits		6.9.2 Couette Flow	
7.9.2 Flow around Immersed Bodies		6.9.3 Steady, Laminar Flow in Circular Tubes	
7.9.3 Flow with a Free Surface		6.9.4 Steady, Axial, Laminar Flow in an Annulus	
7.10 Similitude Based on Governing Differential Equations		6.10 Other Aspects of Differential Analysis	
7.11 Chapter Summary and Study Guide		6.10.1 Numerical Methods	
		Chapter Summary and Study Guide	
Chapter 8 Viscous Flow in Pipes			
8.1 General Characteristics of Pipe Flow	401-415	8.3 Fully Developed Turbulent Flow	418-429
8.1.1 Laminar of Turbulent Flow		8.3.1 Transition from Laminar to Turbulent Flow	
8.1.2 Entrance Region and Fully Developed Flow		8.3.2 Turbulent Shear Stress	
8.1.3 Pressure and Shear Stress		8.3.3 Turbulent Velocity Profile	
8.2 Fully Developed Laminar Flow		8.3.4 Turbulent Modeling	
8.2.1 From F = ma Applied Directly to a Fluid Mechanics Element		8.3.5 Chaos and Turbulence	
8.2.2 From the Navier-Stokes Equations			
8.2.3 From Dimensional Analysis			

Table 4A. Calculus Based Fluid Mechanics Topics for Post-Secondary Engineering Education (Continued)

Chapter/Section	Page Nos.	Chapter/Section	Page Nos.
Chapter 9 Flow over Immersed Bodies		**Chapter 10 Open Channel Flow**	
9.2 Boundary Layer Characteristics	493-518	10.3 Energy Considerations 10.3.2 Channel Depth Variations	573-574
9.2.1 Boundary Layer structure and Thickness on a Flat Plate			
9.2.2 Prandtl/Blasius Boundary Layer Solution			
9.2.3 Momentum Integral boundary Layer Equation for a Flat Plate			
9.2.4 Transition from Laminar to Turbulent Flow			
9.2.5 Turbulent Boundary Layer Flow			
9.2.6 Effects of Pressure Gradient			
9.2.7 Momentum Integral Boundary Layer Equation with Nonzero Pressure Gradient			
Chapter 11 Compressible Flow			
11.1 Ideal Gas Relationships	614-623		
11.2 Mach Number and Speed of Sound			
11.4 Isentropic Flow of an Ideal Gas	628-631		
11.4.1 Effect of Variations in Flow Cross-Sectional Areas			
11.5 Nonisentropic Flow of an Ideal Gas	647-664		
11.5.1 Adiabatic Constant Area Duct Flow with Friction (Fanno Flow)			
11.5.2 Frictionless Constant Area Duct Flow with Heat Transfer (Rayleigh Flow)			

Table 4B. Pre-Requisite Math and Science Topics to Be Reviewed Before Teaching the Calculus Portion of Fluid Mechanics Topics

Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) **[Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)**	
Math	**Physics/Chemistry**
1. [absolute value] (M7N1) → 7th (2A) 2. [analytic geometry] → Post-secondary 3. [analytic geometry: hyperbolic tangent] Post-secondary → To be taught 4. [areas of geometric shapes: circle, triangle, etc.] (M5M1) → 5th (2B) 5. [coordinate system] (M4G3) → 4th (2B) 6. [cross product] → To be taught as a special math topic 7. [cylinder] (M1G1) (M1G2) → 1st (2B) 8. [derivative] → 12th and [partial derivative] → Post-Secondary 9. [dot product] → To be taught as a special math topic 10. [ellipse] (MA2G4) → 10th (2F) → To be taught 11. [Eulerian method] → Post-secondary 12. [exponent] (M6A3) → 6th (2A) 13. [four operations] (M1N3) → 1st (2A) 14. [functions] (MA1A1) → 9th (2E) and others → Post-secondary 15. [gradient "del"] → Post-Secondary 16. [graph] (S7CS6) → 7th (6) 17. [height] (MKM1) → K (2B) 18. [integration] → 12th (To be taught as a special skill) 19. [Lagrangian method] → Post-secondary 20. [limit] → Post-secondary 21. [logarithmic functions] (MA2A5) → 10th (2E) 22. [perimeter] (M3M3) (M3M4) → 3rd (2B) 23. [Pythagorean Theorem] (M8G2) → 8th (2B) 24. [prism] (M6G2) → 6th (2B) 25. [radius] (M3G1) → 3rd (2B) 26. [ratio] (M6A1) → 6th (2A) 27. [sigma notation] (M6N1) → 6th (1A) or (MA1A3) → 9th (2E) 28. [square root] (M8N1) → 8th (2A) 29. [surface] (M6M4) → 6th (2B) 30. [3rd order non-linear differential equation] → Post-secondary 31. [triangle] (M5M1) → 5th (2B) and [trigonometric functions] (MA2G2) → 10th (2F) 32. [unit conversion] (M6M1) → 6th (2C) 33. [vector] (MA3A10) → 11th (2H) → To be taught as a special math topics 34. [volume] (M5M4) → 5th (1B) (M6M3) → 6th (2B) (MA1G5) → 9th (2F)	1. **[absolute temperature]** (SP3) → 9th (3B) → To be taught 2. [acceleration] (S8P3) → 8th (3C) 3. [Dimensional Analysis] → Special topics from 7.1 to be taught 4. [density] (S6E5) → 6th (4A) 5. [energy] (SP3) → 9th (3B) 6. [entropy] → Post-secondary → To be taught 7. [1st moment of the area] → To be taught 8. [force] (S4P3) → 4th (3A) or (S8P3) → 8th (3C) 9. [friction] (S8P3) → 8th (3A) → To be taught 10. **[gas/liquid]** (SPS5) → 9th (3B) → To be taught 11. [graph] (S7CS6) → 7th (6) 12. [gravity] (S6E1) → 6th (3A) 13. [heat] (S2P2) → 2nd (3A) 14. **[Ideal Gas Law]** → Post-secondary → to be taught 15. **[intermolecular cohesive force]** → To be taught 16. [mass] (S8P3) → 8th (3A) 17. [molecule] (S8P1)→ 8th (4A) 18. [momentum] (SP3) → 9th (3B) 19. [Newton's 1st, 2nd and 3rd Laws] (SP1) → 9th (3C) → To be taught 20. [potential energy] (SP3) → 9th (3A) 21. [power] (SP3) → 9th (3B) 22. [pressure] (SC5) → 9th (4B) → To be taught 23. [Reynolds Number] → To be taught as special topic 24. [2nd moment of the area] → To be taught 25. [speed] (S2P3) → 2nd (3A) 26. [speed of sound] (SPS9) → 9th (3B) → To be taught 27. [stress] → To be taught 28. [temperature] (S3P1) → 3rd (3A) and (SP3) → 9th (3B) 29. [torque] → Post-secondary → To be taught 30. [velocity] (S8P3)→ 8th (3A) 31. [wave] (S8P4) → 8th (3A) 32. [weight] (MKM1) → K (2C) 33. [work] (S8P3) → 8th (3A)

Conclusions and Recommendations

This report has presented (1) information about one popular college-level fluid mechanics textbook (the first textbook or the "primary source of data) selected for the initial determination and selection of high school age-possible topics (Table 1), and (2) the outcomes of the research on the inclusion of mathematics, physics and chemistry concepts and skills needed for reading and homework assignments (Tables 2 through 4B). The following are recommended: (1) **Pilot study**: High schools could conduct pilot pedagogic experiments to determine the actual age-feasibility and age-appropriateness of all pre-calculus level fluid mechanics-related analytic knowledge content identified in Table 3 (Pre-Calculus Based Fluid Mechanics Topics That Possibly Could Be Taught at 9^{th} Grade), and Table 4A (Pre-Requisite Mathematics and Science Topics to Be Reviewed Before Teaching the Pre-Calculus Portion of Fluid Mechanics Topics to 9^{th} Grade Students), using the selected textbook; and K-12 mathematics and science teachers could use the same Tables as references to incorporate fluid mechanics topics into respective curriculum; and (2) **Curriculum development**: Existing K-12 engineering and technology curriculum developers could use the same Tables as references for the development of new K-12 engineering instructional materials or for the incorporation of fluid mechanics-related knowledge and skills into their previously developed instructional materials.

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