Engineering Analytic Principles and Predictive Computational Skills for K-12 Students:

Presenting a List of High School 9th Grade Age-Possible Fluid Mechanics Topics to Engineering and Technology Educators and Curriculum Developers

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Introduction

Rationale for Presenting this List

Problem in the current K-12 engineering curriculum

In my previous article titled *Engineering Analytic Principles and Predictive Computational Skills for K-12 Students: Presenting a List of High School 9th Grade Age-Possible Statics Topics to Engineering and Technology Educators and Curriculum Developers*, I have explained one of the problems in the currently predominant practice of K-12 engineering curriculum, which had previously explored by Smith and Wicklein (2007, pp. 2-3) as the "fragmented focus and lack of a clear curriculum framework" which is "detrimental to the potential of the field and have hindered efforts aimed at achieving the stated goals of technological literacy for all students;" and confirmed by an authoritative report issued on September 8, 2009, by the Committee on K-12 Engineering Education established by the National Academy of Engineering and the National Research Council, titled *Engineering in K-12 Education: Understanding the Status and Improving the Prospects* (2009), as the "absence of a clear description of which engineering knowledge, skills, and habits of mind are most important, how they relate to and build on one another, and how and when (i.e., at what age) they should be introduced to students" (2009, pp. 7-8; p. 151).

Conceptual framework for the solution of the problem

In my previous article, I have presented a practical conceptual framework for the definition of defining K-12 age-possible engineering analytic knowledge content, using a four-step procedure; which has been used again for the subject of fluid mechanics:

(1) Select textbooks and student's solution manual that are among the most popular ones for undergraduate engineering fluid mechanics course;

(2) Read carefully every paragraph in the body text to find and record the pre-requisite science knowledge content needed for each topic (notably physics and chemistry).

(3) Find the relevant computational formulas to determine and record the mathematics skills needed. Practically speaking, every engineering topic includes mathematically-based formulas or equations, which shall reveal the level of mathematics required for students to comfortably learn the topic's analytic principles and formula-based predictive computational skills.

(4) Compare the recorded data, i.e., mathematics and science pre-requisites, with the mandates of the Performance Standards for Mathematics and Sciences of the Department of Education of a selected state in the Southern part of the United States, to determine the Grade level for the inclusion of the topic.

Objective of Presenting this List

This *List* is intended to be an "initial list" of high school 9th Grade "age-possible" fluid mechanics topics; whether these topics are actually age-feasible or age-appropriate could be determined only after actual pedagogic experiment or pilot studies have been conducted and analyzed. However, the presentation of this *List* could constitute the critical first step for the extensive integration of fluid mechanics-related engineering analytic principles and predictive computational skills into a viable K-12 engineering and technology curriculum, in a rational, systemic and cohesive way.

Hopefully, the presentation of this *List* could help improve engineering education in the United States, with the following practical applications:

(1) K-12 engineering curriculum development: Current K-12 engineering and technology curriculum developers and teachers, in their endeavors to integrate engineering analytic principles and predictive skills into K-12 engineering and technology curriculum, in a cohesive and systematic way, could use this *List* as a reference in the selection of fluid mechanics topics from the main textbooks listed in Table 1, for pedagogic experiment or pilot study aimed at determining if the topics included in the *List* are indeed age-feasible or age-appropriate for high school 9th Grade students.

(2) Engineering education: K-12 engineering teachers as well as university undergraduate engineering professors could use the *List* as a reference to review pertinent mathematics skills and scientific principles at the start of engineering courses with their students, for the statics topics that require only pre-calculus mathematics skills.

(3) K-12 mathematics and science education: K-12 teachers could use this *List* as a reference to create extra learning materials focused on the applications of mathematics skills and scientific principles in engineering, and thus, help students to understand the relevance of mathematics skills and scientific principles to practical solution of engineering design problems.

Source of Data

University undergraduate fluid mechanics textbooks and learning materials that have been used as data source in the research are shown in Table 1 and *Figure 1*.

Table 1. Textbook Information

	Main Textbook	**Reference Book**	**Student Solution Manual**
Title	Fundamentals of Fluid Mechanics Mechanics, 5th Edition	A Brief Introduction to Fluid Mechanics, 4th Edition	A Brief Introduction to Fluid Mechanics, Student Solutions Manual, 4th Edition
Authors	Bruce M. Munson, Donald F. Young, Theodore H. Okiishi	Donald F. Young, Bruce R. Munson, Theodore H. Okiishi, Wade W. Huebsch	Donald F. Young, Bruce R. Munson, Theodore H. Okiishi, Wade W. Huebsch
Publisher	John Wiley & Sons, Inc.	John Wiley & Sons, Inc.	John Wiley & Sons, Inc.
Year	2006	2007	2007
ISBN	0-471-67582-2	978-0470039625	978-0470099285
Application	Used for the extraction of fluid mechanics related engineering analytic/predictive principles and computational formulas (the main textbook).	Used as a reference book.	Used to double-check for the mathematics and physics principles and computational skills needed for the study of various topics of fluid mechanics contained in the main textbook.

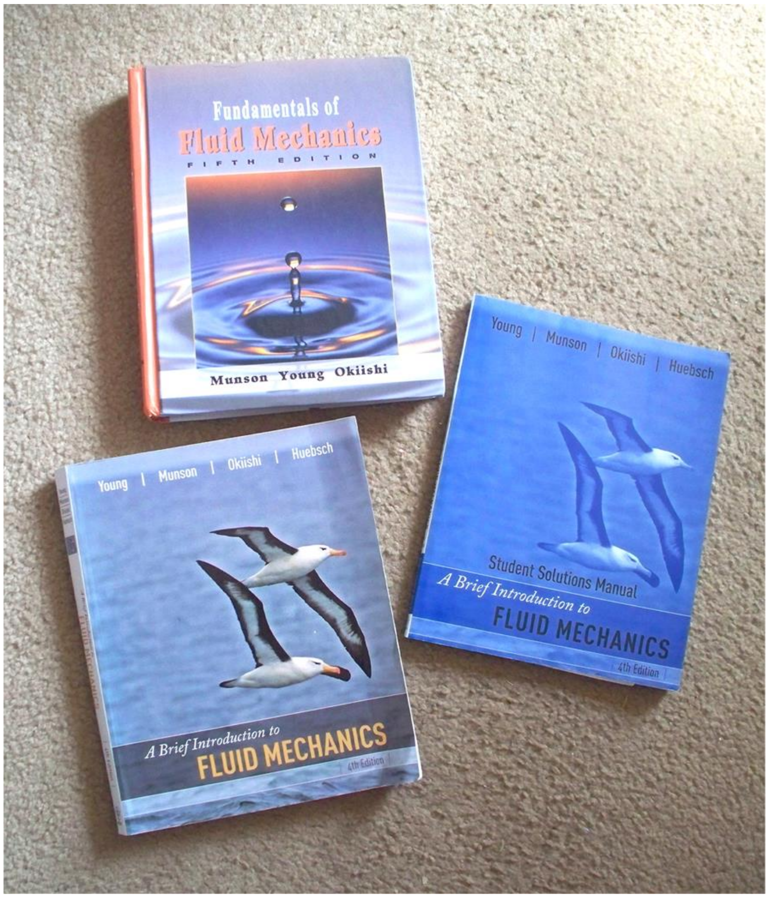

Figure 1. University undergraduate fluid mechanics textbooks and Student's Solutions Manuals used in the research as data source (used at California State University, Los Angeles).

Outcomes of the Research

The outcome of this research is very encouraging. A substantial amount of engineering declarative knowledge content covered in the selected university undergraduate fluid mechanics textbook has been initially determined to be pedagogically possible for 9th Grade high school students, based on the mandates of the Mathematics and Science Performance Standards of a selected state in the United States.

Initial Determination of High School Age-Possible Fluid Mechanics Topics

Table 2 constitutes the *Initial List of High School 9th Grade Age-Possible Fluid Mechanics Topics*, and is the centerpiece of this article. *Figure 2* illustrate how to use this *List*.

The statistic summary of the research project (Table 4) indicates that a significant portion of fluid mechanics knowledge content covered in the selected undergraduate level textbook could possibly be taught to high school students at 9th Grade. 62.2% of all sections, and 51.0% of the volume in the selected main textbook is based on pre-calculus mathematics and on principles of physics students are supposed to learn before or by 9th

Grade, according to the Mathematics and Science Performance Standards of the selected state's Department of Education.

Initial Determination of Pre-Requisite Mathematics and Science Topics

Table 3 constitutes the *Pre-Requisite Mathematics and Science Topics to Be Reviewed before Teaching the Pre-Calculus Portion of Fluid Mechanics Topics to 9th Grade Students*. This list includes 24 sets of mathematics principles and skills, as well as 28 sets of physics and chemistry principles and skills that are needed as pre-requisites or as important topics to be reviewed for the effective learning of fluid mechanics topics initially determined as age-possible for 9th Grade students.

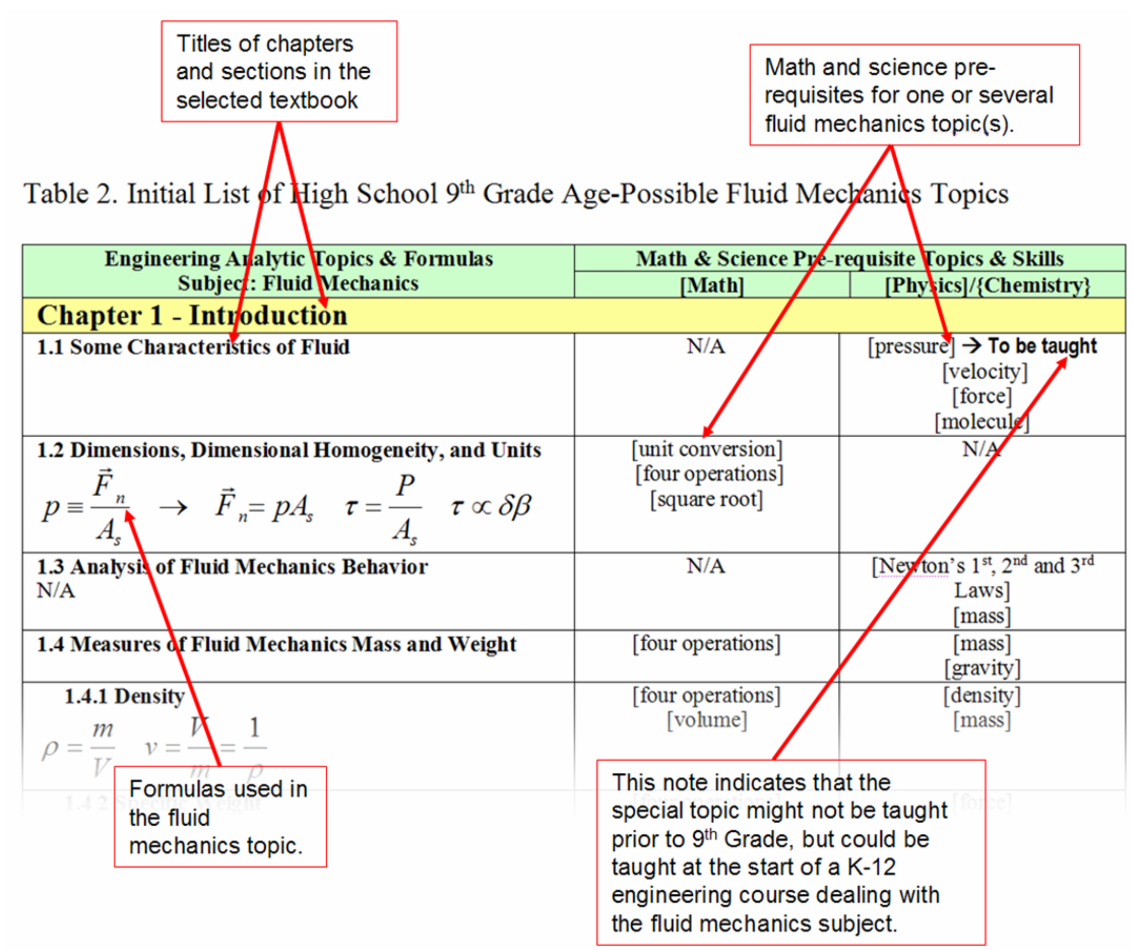

Table 2. Initial List of High School 9th Grade Age-Possible Fluid Mechanics Topics

Engineering Analytic Topics & Formulas Subject: Fluid Mechanics	Math & Science Pre-requisite Topics & Skills [Math]	[Physics]/{Chemistry}
Chapter 1 - Introduction		
1.1 Some Characteristics of Fluid	N/A	[pressure] → **To be taught** [velocity] [force] [molecule]
1.2 Dimensions, Dimensional Homogeneity, and Units $p \equiv \frac{\vec{F}_n}{A_s} \rightarrow \vec{F}_n = pA_s \quad \tau = \frac{P}{A_s} \quad \tau \propto \delta\beta$	[unit conversion] [four operations] [square root]	N/A
1.3 Analysis of Fluid Mechanics Behavior N/A	N/A	[Newton's 1st, 2nd and 3rd Laws] [mass]
1.4 Measures of Fluid Mechanics Mass and Weight	[four operations]	[mass] [gravity]
1.4.1 Density $\rho = \frac{m}{V} \quad v = \frac{V}{m} = \frac{1}{\rho}$	[four operations] [volume]	[density] [mass]

Figure 2. The Initial List of High School 9th Grade Age-Possible Fluid Mechanics Topics.

Conclusions and Recommendations

This article has provided (1) a reference list for high school 9th Grade age-possible fluid mechanics topic, and (2) a reference list for the review of mathematics and science pre-requisites. In order to improve K-12 engineering education, the following recommendations and plans are hereby presented for consideration, support and implementation:

1. Pilot study: K-12 schools (especially high schools, including chartered high schools) could be found to conduct pilot pedagogic experiments to determine the age-feasibility and age-appropriateness of all fluid mechanics-related analytic knowledge content identified in *Initial List of High School 9th Grade Age-Possible Fluid Mechanics Topics* (Table 2). Likewise, K-12 mathematics and science teachers could use the same *List* as a reference to incorporate pertinent fluid mechanics topics in their respective curriculum.
2. Change within the system: We could encourage existing K-12 engineering and technology curriculum developers to use the same *List* as a reference to incorporate fluid mechanics-related engineering knowledge and skills into their previously developed instructional materials, or to create new ones.

References

Committee on K-12 Engineering Education (2009). *Engineering in K-12 education: Understanding the status and improving the prospects*. Washington, DC: National Academy of Engineering and the National Research Council.

Smith, P. C., & Wicklein, R. C. (2007). *Identifying the essential aspects and related academic concepts of an engineering design curriculum in secondary technology education.* Unpublished internal research report, NCETE. Retrieved January 30, 2009 from http://ncete.org/flash/publications.php

Table 2. Initial List of High School 9th Grade Age-Possible Fluid Mechanics Topics

Engineering Analytic Topics & Formulas **Subject: Fluid Mechanics**	**Math & Science Pre-requisite Topics & Skills** **[Math]**	 **[Physics]/{Chemistry}**
Chapter 1 - Introduction		
1.1 Some Characteristics of Fluid	N/A	[pressure] → **To be taught** [velocity] [force] [molecule]
1.2 Dimensions, Dimensional Homogeneity, and Units $p \equiv \frac{\vec{F}_n}{A_s} \quad \rightarrow \quad \vec{F}_n = pA_s \quad \tau = \frac{P}{A_s} \quad \tau \propto \delta\beta$	[unit conversion] [four operations] [square root]	N/A
1.3 Analysis of Fluid Mechanics Behavior N/A	N/A	[Newton's 1st, 2nd and 3rd Laws] [mass]
1.4 Measures of Fluid Mechanics Mass and Weight	[four operations]	[mass] [gravity]
1.4.1 Density $\rho = \frac{m}{V} \quad v = \frac{V}{m} = \frac{1}{\rho}$	[four operations] [volume]	[density] [mass]
1.4 2 Specific Weight $\gamma \equiv \frac{W}{V} = \frac{mg}{V} = \rho g$	[four operations]	[force] [gravity] [density]
1.4.3 Specific Gravity $SG = \frac{\rho}{\rho_{H_2O} @ 4^\circ C}$		[pressure] → **To be taught**
1.5 Ideal Gas Law $p = \rho RT$		[temperature] {**absolute temperature**} → **To be taught** [density]
1.7 Compressibility of Fluids	N/A	N/A
1.7.2 Compression and Expansion of Gases $\frac{p}{\rho} = \text{Constant} \quad \frac{p}{\rho^k} = \text{Constant} \quad E_v = p \quad E_v = kp$	[four operations] [exponent]	[pressure] → **To be taught** [density]
1.8 Vapor Pressure	N/A	**{intermolecular cohesive force}** → **To be taught** [momentum] [pressure] → **To be taught**
1.9 Surface Tension $2\pi R\sigma = \Delta p \pi R^2 \quad \Delta p = p_i - p_e = \frac{2\sigma}{R}$ $\gamma \pi R^2 h = 2\pi R\sigma \cos\theta \quad \rightarrow \quad h = \frac{2\sigma\cos\theta}{\gamma R}$	[areas of geometric shapes: circle, triangle] [unit conversion] [height] [trigonometric functions]	[force] [mass] [pressure] → **To be taught** [weight] [gravity]
1.10 A Brief Look Back in History	N/A	N/A
1.11 Chapter Summary and Study Guide		

Table 2. (Continued).

Engineering Analytic Topics & Formulas Subject: Fluid Mechanics	Math & Science Pre-requisite Topics & Skills [Math]	 [Physics]/{Chemistry}
Chapter 2 Fluid Statics		
2.3.1 Incompressible Fluid $\int_{p_1}^{p_2} dp = -\gamma \int_{z_1}^{z_2} dz \rightarrow \begin{cases} p_2 - p_1 = -\gamma(z_2 - z_1) \\ p_1 - p_2 = \gamma(z_2 - z_1) \end{cases}$ $\rightarrow \quad p_1 - p_2 = \gamma h \quad \rightarrow \begin{cases} p_1 = \gamma h + p_2 \\ h = \dfrac{p_1 - p_2}{\gamma} \end{cases}$ $p = \gamma h + p_0 = \rho g h + p_0$	[four operations] [integration] → **Post-Secondary** **Note: The main Formula** $p = \gamma h + p_0 = \rho g h + p_0$ **does not need calculus. The calculus-based formulas could be removed.**	[pressure] → **To be taught**
2.3.2 Compressible Fluid $\left.\begin{matrix} p = \rho R T \\ \rho = \dfrac{p}{RT} \\ \dfrac{dp}{dz} = -\gamma = -\rho g \end{matrix}\right\} \dfrac{dp}{dz} = -\dfrac{gp}{RT} \rightarrow$ $(dz)\dfrac{dp}{dz(p)} = -\dfrac{gp}{RT(p)}(dz) \quad \dfrac{dp}{p} = -\dfrac{g}{RT}dz \rightarrow$ $\int_{p_1}^{p_2} \dfrac{dp}{p} = \int_{z_1}^{z_2} -\dfrac{g}{RT}dz = -\dfrac{g}{R}\int_{z_1}^{z_2}\dfrac{dz}{T} \rightarrow$ $\int_{p_1}^{p_2} \dfrac{dp}{p} = \ln\dfrac{p_2}{p_1} = -\dfrac{g}{R}\int_{z_1}^{z_2}\dfrac{dz}{T}$ $p_2 = p_1 \exp\left[-\dfrac{g(z_2 - z_1)}{RT_0}\right]$	[four operations] [exponent] [integration] → **Post-Secondary** [derivative] → **Post-Secondary** **Note: The main formula** $p_2 = p_1 \exp\left[-\dfrac{g(z_2 - z_1)}{RT_0}\right]$ **does not need calculus. The calculus-based formulas could be removed.**	[pressure] → **To be taught** {**absolute temperature**} → **To be taught** **{gas/liquid}** → **To be taught**
2.4 Standard Atmosphere $T = T_a - \beta z \quad p = p_a\left(1 - \dfrac{\beta z}{T_a}\right)^{g/R\beta}$	[four operations] [exponent]	[temperature] [pressure] → **To be taught** [density] [weight]
2.5 Measurement of Pressure $p_{abs} = p_{gage} + p_{atm} \quad p_{atm} = \gamma h + p_{vapor}$	[four operations]	[pressure] → **To be taught**
2.6 Monometry	[four operations] [cylinder]	
2.6.1 Piezometer Tube $p = \gamma h + p_0 \quad p_A = \gamma_1 h_1$	[four operations] [height]	
2.6.2 U-Tube Manometer $p = \gamma h + p_0 \quad p_A + \gamma_1 h_1 - \gamma_2 h_2 = 0 \quad \rightarrow$ $p_A = \gamma_2 h_2 - \gamma_1 h_1 \quad p_A = \gamma_2 h_2$ $p_A + \gamma_1 h_1 - \gamma_2 h_2 - \gamma_3 h_3 = p_B \quad \rightarrow$ $p_A - p_B = \gamma_2 h_2 + \gamma_3 h_3 - +\gamma_1 h_1$	[four operations]	
2.6.3 Inclined-Tube Manometer $p_A + \gamma_1 h_1 - \gamma_2 \ell_2 \sin\theta - \gamma_3 h_3 = p_B$ $p_A - p_B = \gamma_2 \ell_2 \sin\theta + \gamma_3 h_3 - \gamma_1 h_1$ $p_A - p_B = \gamma_2 \ell_2 \sin\theta \quad \rightarrow \quad \ell_2 = \dfrac{p_A - p_B}{\gamma_2 \sin\theta}$	[four operations] [trigonometric functions]	
2.7 Mechanical and Electronic Pressure Measuring Devices	N/A	N/A

Table 2. (Continued).

Engineering Analytic Topics & Formulas **Subject: Fluid Mechanics**	**Math & Science Pre-requisite Topics & Skills** **[Math]**	 **[Physics]/{Chemistry}**
Chapter 2 Fluid Statics (Continued)		
2.9 Pressure Prism $F_R = p_{av}A = \gamma\left(\frac{h}{2}\right)A \quad F_R = volume = \frac{1}{2}(\gamma h)(bh) = \gamma\left(\frac{h}{2}\right)A$ $F_R = F_1 + F_2 \quad F_R y_A = F_1 y_1 + F_2 y_2$	[four operations] [prism]	[pressure] → **To be taught** [force]
2.10 Hydrostatic Force on a Curves Surface $F_H = F_2 \quad F_v = F_1 + \vec{W}$ $F_R = \sqrt{(F_H)^2 + (F_v)^2}$	[four operations] [Pythagorean Theorem]	[force]
2.11 Buoyancy, Flotation, and Stability N/A **2.11.1 Archimedes' Principle** $F_B = F_2 - F_1 - \vec{W} \quad F_2 - F_1 = \gamma(h_2 - h_1)A$ $F_B = \gamma(h_2 - h_1)A - \gamma[(h_2 - h_1)A - \mathcal{V}]$ $F_B = \gamma\mathcal{V} \quad F_B y_c = F_2 y_1 - F_1 y_1 - \vec{W} y_2$ $\mathcal{V} y_c = \mathcal{V}_T y_1 - (\mathcal{V}_T - \mathcal{V})y_2$ **2.11.2 Stability** N/A	[four operations] [volume]	[force] [weight]
2.13 Chapter Summary and Study Guide	N/A	N/A
Chapter 3 Elementary Fluid Dynamics – The Bernoulli Equation		
3.1 Newton's Second Law $\vec{F} = m\vec{a} \quad \sum(\vec{F}_P + \vec{F}_g) = m\vec{a}$ $a_s = V\frac{\partial V}{\partial s} \quad a_n = V\frac{V^2}{\Re} \quad \leftarrow \quad V = \lvert\vec{V}\rvert$	[four operations] [partial derivative] → **Post-secondary** [volume] **Note: The main formula** $\vec{F} = m\vec{a}$ **does not need calculus. The calculus-based formulas could be removed.**	[Newton's 1st, 2nd and 3rd Laws] → **To be taught** [force] [speed]
3.2 F = ma along a Streamline $\sum \delta F_s = \delta m a_s = \delta m V\frac{\partial V}{\partial s} = \rho\delta\mathcal{V} V\frac{\partial V}{\partial s}$ $\left.\begin{array}{l}\delta\vec{W} = \gamma\delta\mathcal{V} \\ \gamma = \rho g\end{array}\right\} \rightarrow \quad \delta\vec{W}_s = -\delta\vec{W}\sin\theta = -\gamma\delta X\mathcal{V}\sin\theta$ $\delta p_s \approx \frac{\partial p}{\partial s}\frac{\delta s}{2}$ $\delta F_{ps} = (p - \delta p_s)\delta n\delta y - (p + \delta p_s)\delta n\delta y = -2\delta p_s\delta n\delta y$ $= -\frac{\partial p}{\partial s}\delta s\delta n\delta y = -\frac{\partial p}{\partial s}\delta\mathcal{V}$ $\sum \delta F_s = \delta\vec{W}_s + \delta F_{ps} = \left(-\gamma\sin\theta - \frac{\partial p}{\partial s}\right)\delta\mathcal{V}$ $-\gamma\sin\theta - \frac{\partial p}{\partial s} = \rho V\frac{\partial V}{\partial s} = \rho a_s$ $-\gamma\frac{dz}{ds} - \frac{dp}{ds} = \frac{1}{2}\rho\frac{d(V^2)}{ds} \quad \rightarrow \quad dp + \frac{1}{2}\rho d(V^2) + \gamma\ dz = 0$	[four operations] [trigonometric functions] [partial derivative] → Post-secondary [sigma notation] **Note: The main formulas** $\vec{F} = m\vec{a}$ and $p + \frac{1}{2}\rho V^2 + \gamma z$ = constant along a streamline (Bernoulli Equation) **do not need calculus. The calculus-based formulas could be removed.**	[force] [gravity] [mass] [acceleration]

Table 2. (Continued).

Engineering Analytic Topics & Formulas **Subject: Fluid Mechanics**	**Math & Science Pre-requisite Topics & Skills** **[Math]**	**[Physics]/{Chemistry}**
Chapter 3 Elementary Fluid Dynamics – The Bernoulli Equation (Continued)		
3.2 F = ma along a Streamline (Continued) $\int \frac{dp}{\rho} + \frac{1}{2}V^2 + gz = C$ (along a streamline) $p + \frac{1}{2}\rho V^2 + \gamma z = \text{constant along a streamline}$ (Bernoulli Equation)	↑ Refer to the previous page.	↑ Refer to the previous page.
3.4 Physical Interpretation $p + \frac{1}{2}\rho V^2 + \gamma z = \text{Constant along the streamline}$ $p + \rho\int \frac{V^2}{\Re} dn + \gamma z = \text{constant across the streamline}$ $\frac{p}{\gamma} + \frac{V^2}{2g} + z = \text{constant on a streamline}$	[four operations] [integration] → **Past-Secondary** **Note: The main formula** $p + \frac{1}{2}\rho V^2 + \gamma z$ $= \text{Constant along the streamline}$ $\frac{p}{\gamma} + \frac{V^2}{2g} + z$ $= \text{constant on a streamline}$ **do not need calculus. The calculus-based formulas could be removed.**	[density] [speed] [gravity]
3.5 Static, Stagnation, Dynamic, and Total Pressure $p_2 = p_1 + \frac{1}{2}\rho V_1^2$ $p + \frac{1}{2}V^2 + \gamma z = p_T = \text{constant along a streamline}$ $\left.\begin{array}{l} p_3 = p + \frac{1}{2}\rho V^2 \\ p_4 = p_1 = p \end{array}\right\} \rightarrow p_3 - p_4 = \frac{1}{2}\rho V^2$ $V = \sqrt{\frac{2(p_3 - p_4)}{\rho}}$	[four operations] [square root]	[pressure] → **To be taught** [density] [speed]
3.6 Examples of Use of the Bernoulli Equation $p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$	[four operations] [exponent]	[pressure] → **To be taught** [density] [speed]
3.6.1 Free Jets $\gamma h = \frac{1}{2}\rho V^2 \rightarrow \begin{cases} V = \sqrt{2\frac{\gamma h}{\rho}} = \sqrt{2gh} \\ V = \sqrt{2g(h+H)} \end{cases}$	[four operations] [exponent] [square root]	
3.6.2 Confined Flows $\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \rightarrow A_1 V_1 = A_2 V_2 \rightarrow Q_1 = Q_2$	[four operations] [exponent] [areas of geometric shapes]	[density] [speed]
3.6.3 Flowrate Measurement $p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2 \quad Q = A_1 V_1 = A_2 V_2$	[four operations] [exponent] [square root] [areas of geometric shapes]	[density] [speed] [pressure] → **To be taught** [gravity]

Table 2. (Continued).

Engineering Analytic Topics & Formulas **Subject: Fluid Mechanics**	**Math & Science Pre-requisite Topics & Skills**	
	[Math]	**[Physics]/{Chemistry}**
Chapter 3 Elementary Fluid Dynamics – The Bernoulli Equation (Continued)		
3.6.3 Flowrate Measurement (Continued) $Q = A_2\sqrt{\frac{2(p_1-p_2)}{\rho\left[1-\left(\frac{A_2}{A_1}\right)^2\right]}}$ $p_1+\frac{1}{2}\rho V_1^2+\gamma z_1 = p_2+\frac{1}{2}\rho V_2^2+\gamma z_2$ $Q = A_1V_1 = bV_1z_1 = A_2V_2 = bV_2z_2$ $p_1 = p_2 = 0 \quad\rightarrow\quad Q = z_2 b\sqrt{\frac{2g(z_1-z_2)}{1-\left(\frac{z_2}{z_1}\right)^2}}$ $Q = C_1Hb\sqrt{2gH} = C_1b\sqrt{2g}H^{3/2}$ $z_1 >> z_2 \quad\rightarrow\quad Q = z_2b\sqrt{2gz_1}$	↑ Refer to the previous page.	↑ Refer to the previous page.
3.7 The Energy Line and the Hydraulic Grade Line $\frac{p}{\gamma}+\frac{V^2}{2g}+z = \text{constant on a streamline} = H$	[four operations] [exponent] [square root]	[density] [speed] [gravity]
3.8 Restrictions on Use of the Bernoulli Equation **3.8.1 Compressibility Effects** $RT\int\frac{dp}{p}+\frac{1}{2}V^2+gz = \text{constant} \qquad \rho = \frac{p}{RT}$ $\frac{V_1^2}{2g}+z_1+\frac{RT}{g}\ln\left(\frac{p_1}{p_2}\right) = \frac{V_2^2}{2g}+z_2$ $C^{1/k}\int p^{-1/k}dp+\frac{1}{2}V^2+gz = \text{constant}$ $C^{1/k}\int_{p_1}^{p_2} p^{-1/k}dp = C^{1/k}\left(\frac{k}{k-1}\right)\left[p_2^{(k-1)/k}-p_1^{(k-1)/k}\right]$ $=\left(\frac{k}{k-1}\right)\left(\frac{p_2}{\rho_1}-\frac{p_1}{\rho_1}\right)$ $\left(\frac{k}{k-1}\right)\frac{p_1}{\rho_1}+\frac{V_1^2}{2}+gz_1 = \left(\frac{k}{k-1}\right)\frac{p_2}{\rho_2}+\frac{V_2^2}{2}+gz_2$ $\frac{p_2-p_1}{p_1} = \left[\left(1+\frac{k-1}{2}Ma_1^2\right)^{k/k-1}-1\right]$ (compressible) $\left.\begin{array}{l}\frac{\rho_2}{\rho_1} = \frac{V_1^2}{2RT_1}\\ Ma_1 = \frac{V_1}{\sqrt{kRT_1}}\end{array}\right\} \rightarrow \frac{p_2-p_1}{p_1} = \frac{kMa_1^2}{2}$ (incompressible) $\frac{p_2-p_1}{p_1} = \frac{kMa_1^2}{2}\left(1+\frac{1}{4}Ma_1^2+\frac{2-k}{24}Ma_1^4+\ldots\right)$ (compressible)	[four operations] [exponent] [logarithmic functions] → **To be taught or reviewed as a special skill** [integration] → **Post-Secondary** **Note: The main formulas** $\rho = \frac{p}{RT}$ **And some others do not need calculus.** **The calculus-based formulas could be removed.**	

Table 2. (Continued).

Engineering Analytic Topics & Formulas Subject: Fluid Mechanics	Math & Science Pre-requisite Topics & Skills [Math]	 [Physics]/{Chemistry}
Chapter 3 Elementary Fluid Dynamics – The Bernoulli Equation (Continued)		
3. 8.3 Rotational Effects $p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2 = \text{constant} = C_{12}$ $\left.\begin{array}{l} V_1 = V_2 = V_0 \\ z_1 = z_2 = 0 \\ p_1 = p_2 = p_0 \end{array}\right\} \rightarrow C_{12} = \frac{1}{2}\rho V_0^2 + p_0$ $\left.\begin{array}{l} V_3 = V_4 = V_0 \\ z_3 = z_4 = h \\ \vec{F} = m\vec{a} \\ p_3 = p_1 - \gamma h \\ p_3 = p_4 \end{array}\right\} \rightarrow C_{34} = C_{12} \rightarrow$ $p + \frac{1}{2}\rho V^2 + \gamma z = \text{constant throughout flow}$ $p_4 = p_5 + \gamma H = \gamma H \quad H = \frac{p_4}{\gamma}$ **3.8.4 Other Restrictions** **3.9 Chapter Summary and Study Guide**	[four operations]	[pressure] → **To be taught** [density] [speed]
Chapter 4 Fluid Kinematics		
4.3 Control Volume and System Representations $F = \frac{d(mv)}{dt}$	[four operations] [volume] [areas of geometric shapes]	[velocity]
4.4 The Reynolds Transport Theorem $B = mb \quad \begin{cases} B = m \rightarrow b = 1 \\ B = \frac{mV^2}{2} \rightarrow b = \frac{V^2}{2} \\ \vec{B} = m\vec{V} \rightarrow \vec{b} = \vec{V} \end{cases}$ B : Extensive Property b : Intensive Property Infinitesimal fluid particles : $\delta V \rightarrow 0$	[four operations] [integration] → **Post-Secondary.** **The calculus-based formulas from 4.4, 4.4.1 to 4.4.6 could be removed.**	[velocity] [acceleration] [mass] [temperature] [momentum]
4.4.7 Selection of a Control Volume N/A	N/A	N/A
4.5 Chapter Summary and study Guide N/A		

Table 2. (Continued).

Engineering Analytic Topics & Formulas Subject: Fluid Mechanics	Math & Science Pre-requisite Topics & Skills	
	[Math]	[Physics]/{Chemistry}
Chapter 5 Finite Control Volume Analysis		
5.1.2 Fixed, Non-deforming Control Volume $\frac{\partial}{\partial t}\int_{cv}\rho\, d\forall \quad \sum \dot{m}_{out} - \sum \dot{m}_{in} = 0 \quad \sum Q_{out} - \sum Q_{in} = 0$ $\frac{\partial}{\partial t}\int_{cv}\rho\, d\forall$ $\dot{m} = \rho AV$ uniformly distributed over the opening in the control surface (one - dimensional flow) $\quad \dot{m} = \rho A\bar{V}$ uniformly distributed over the opening in the control surface $\dot{m} = \rho_1 A_1 \bar{V}_1 = \rho_2 A_2 \bar{V}_2 \quad Q = A_1\bar{V}_1 = A_2\bar{V}_2$ $\sum \dot{m}_{in} = \sum \dot{m}_{out}$	[four operations] [analytic geometry] → **12th (To be taught or reviewed as a special skill)** [volume] [areas of geometric shapes] [integration] → **12th (To be taught or reviewed as a special skill)** [sigma notation] **Note: The main formula** $\dot{m} = \rho_1 A_1 \bar{V}_1 = \rho_2 A_2 \bar{V}_2$ $Q = A_1\bar{V}_1 = A_2\bar{V}_2$ $\sum \dot{m}_{in} = \sum \dot{m}_{out}$ **are not based on calculus. The calculus-based formulas could be removed.**	[mass] [density] [velocity]
5.2 Newton's Second Law – The Linear Momentum and Moment-of-Momentum Equation N/A	N/A	N/A
5.3.3 Comparison of the Energy Equation with the Bernoulli Equation $\dot{m}\left[\check{u}_{out} - \check{u}_{in} + \frac{p_{out}}{\rho} - \frac{p_{in}}{\rho} + \frac{V_{out}^2 - V_{in}^2}{2} + g\left(z_{out} - z_{in}\right)\right] = \dot{Q}_{\substack{net\\in}}$ $\frac{p_{out}}{\rho} + \frac{V_{out}^2}{2} + gz_{out} = \frac{p_{in}}{\rho} + \frac{V_{in}^2}{2} + gz_{in} - \left(\check{u}_{out} - \check{u}_{in} - q_{\substack{net\\in}}\right)$ $q_{\substack{net\\in}} = \frac{\dot{Q}_{net\ in}}{\dot{m}}$ $p_{out} + \frac{\rho V_{out}^2}{2} + \gamma z_{out} = p_{in} + \frac{\rho V_{in}^2}{2} + \gamma z_{in} \quad \gamma = \rho g \quad \rightarrow \quad \frac{\gamma}{\rho} = g$ $\frac{\left(p_{out} + \frac{\rho V_{out}^2}{2} + \gamma z_{out}\right)}{\rho} = \frac{\left(p_{in} + \frac{\rho V_{in}^2}{2} + \gamma z_{in}\right)}{\rho} \quad \rightarrow$ $\frac{p_{out}}{\rho} + \frac{V_{out}^2}{2} + gz_{out} = \frac{p_{in}}{\rho} + \frac{V_{in}^2}{2} + gz_{in}$ $\check{u}_{out} - \check{u}_{in} - q_{\substack{net\\in}} = 0$ (Frictionless steady incompressible flow) $\check{u}_{out} - \check{u}_{in} - q_{\substack{net\\in}} > 0$ (Steady incompressible flow with friction) $\check{u}_{out} - \check{u}_{in} - q_{\substack{net\\in}} = \text{loss}$	[four operations] [areas of geometric shapes] [volume] [derivatives] → **Post-Secondary** **The calculus-based formulas could be removed.**	[velocity] [gravity] [density] [mass]

Table 2. (Continued).

Engineering Analytic Topics & Formulas Subject: Fluid Mechanics	Math & Science Pre-requisite Topics & Skills	
	[Math]	[Physics]/{Chemistry}
Chapter 5 Finite Control Volume Analysis (Continued)		
5.3.3 Comparison of the Energy Equation with the Bernoulli Equation $\frac{p_{out}}{\rho}+\frac{V_{out}^2}{2}+gz_{out}=\frac{p_{in}}{\rho}+\frac{V_{in}^2}{2}+gz_{in}-\text{loss}$ $\dot{m}\left[\check{u}_{out}-\check{u}_{in}+\frac{p_{out}}{\rho}-\frac{p_{in}}{\rho}+\frac{V_{out}^2-V_{in}^2}{2}+g(z_{out}-z_{in})\right]$ $=\dot{Q}_{\underset{in}{net}}+\dot{W}_{\underset{net\ in}{shaft}}$ $\frac{p_{out}}{\rho}+\frac{V_{out}^2}{2}+gz_{out}=\frac{p_{in}}{\rho}+\frac{V_{in}^2}{2}+gz_{in}+w_{\underset{net\ in}{shaft}}-$ $\left(\check{u}_{out}-\check{u}_{in}-q_{\underset{in}{net}}\right)$ $\frac{p_{out}}{\rho}+\frac{V_{out}^2}{2}+gz_{out}=\frac{p_{in}}{\rho}+\frac{V_{in}^2}{2}+gz_{in}+w_{\underset{net\ in}{shaft}}-\text{loss}$ $p_{out}+\frac{\rho V_{out}^2}{2}+\gamma z_{out}=p_{in}+\frac{\rho V_{in}^2}{2}+\gamma z_{in}+\rho w_{\underset{net\ in}{shaft}}-\rho(\text{loss})$ $\frac{\left(\frac{p_{out}}{\rho}+\frac{V_{out}^2}{2}+gz_{out}=\frac{p_{in}}{\rho}+\frac{V_{in}^2}{2}+gz_{in}+w_{\underset{net\ in}{shaft}}-\text{loss}\right)}{g}\rightarrow$ $\frac{p_{out}}{\gamma}+\frac{V_{out}^2}{2g}+z_{out}=\frac{p_{in}}{\gamma}+\frac{V_{in}^2}{2g}+z_{in}+w_{\underset{net\ in}{shaft}}-h_s-h_L$ $h_s=\frac{w_{shaft\ net\ in}}{g}=\frac{\dot{W}_{\underset{net\ in}{shaft}}}{\dot{m}g}=\frac{\dot{W}_{\underset{net\ in}{shaft}}}{\gamma Q}$ $h_T=-(h_s+h_L)_T \quad h_p=(h_s+h_L)_P$	[four operations] [areas of geometric shapes] [volume] [derivatives] → **Post-Secondary**	[velocity] [gravity] [density] [mass]
5.3.4 Application of the Energy Equation to Non-uniform Flow $\int_{cs}\frac{V^2}{2}\rho\vec{V}\cdot\hat{n}\ dA=\dot{m}\left(\frac{\alpha_{out}\bar{V}_{out}^2}{2}-\frac{\alpha_{in}\bar{V}_{in}^2}{2}\right)$ $\frac{\dot{m}\alpha\bar{V}^2}{2}=\int_A\frac{V^2}{2}\rho\vec{V}\cdot\hat{n}\ dA \quad \alpha=\frac{\int_A(V^2/2)\rho\vec{V}\cdot\hat{n}\ dA}{\dot{m}\alpha\bar{V}^2/2}$ $\frac{p_{out}}{\rho}+\frac{\alpha_{out}\bar{V}_{out}^2}{2}+gz_{out}=\frac{p_{in}}{\rho}+\frac{\alpha_{in}\bar{V}_{in}^2}{2}+gz_{in}+w_{\underset{net\ in}{shaft}}-\text{loss}$ $\left(\frac{p_{out}}{\rho}+\frac{\alpha_{out}\bar{V}_{out}^2}{2}+gz_{out}=\frac{p_{in}}{\rho}+\frac{\alpha_{in}\bar{V}_{in}^2}{2}+gz_{in}+w_{\underset{net\ in}{shaft}}-\text{loss}\right)(\rho)$ $\rightarrow$ $p_{out}+\frac{\rho\alpha_{out}\bar{V}_{out}^2}{2}+\gamma z_{out}=p_{in}+\frac{\rho\alpha_{in}\bar{V}_{in}^2}{2}+\gamma z_{in}+\rho w_{\underset{net\ in}{shaft}}-\rho(\text{loss})$ $\frac{\left(\frac{p_{out}}{\rho}+\frac{\alpha_{out}\bar{V}_{out}^2}{2}+gz_{out}=\frac{p_{in}}{\rho}+\frac{\alpha_{in}\bar{V}_{in}^2}{2}+gz_{in}+w_{\underset{net\ in}{shaft}}-\text{loss}\right)}{g}$ $\rightarrow$	[four operations] [areas of geometric shapes] [volume] [integration] → **Post-Secondary** **Note: The main formulas** $\frac{p_{out}}{\gamma}+\frac{\alpha_{out}\bar{V}_{out}^2}{2g}+z_{out}$ $=\frac{p_{in}}{\gamma}+\frac{\alpha_{in}\bar{V}_{in}^2}{2g}+z_{in}$ $+\frac{w_{\underset{net\ in}{shaft}}}{g}-h_L$ **is based on pre-calculus mathematics. Calculus-based formulas used to derive this formula could be removed from 9th Grade classroom instruction.**	[velocity] [gravity] [density] [mass]

Table 2. (Continued).

Engineering Analytic Topics & Formulas Subject: Fluid Mechanics	Math & Science Pre-requisite Topics & Skills [Math]	[Physics]/{Chemistry}
Chapter 5 Finite Control Volume Analysis (Continued)		
5.3.4 Application of the Energy Equation to Non-uniform Flow (Continued) $\frac{p_{out}}{\gamma}+\frac{\alpha_{out}\bar{V}_{out}^2}{2g}+z_{out}=\frac{p_{in}}{\gamma}+\frac{\alpha_{in}\bar{V}_{in}^2}{2g}+z_{in}+\frac{w_{\substack{shaft\\net\ in}}}{g}-h_L$	↑ Refer to the previous page.	↑ Refer to the previous page.
5.3.5 Combination of the Energy Equation and the Moment-of-momentum Equation $\eta=\frac{w_{\substack{shaft\\net\ in}}-\text{loss}}{w_{\substack{shaft\\net\ in}}}$	[four operations]	[heat] [temperature]
5.4.4 Application of the Loss Form of the Energy Equation $\frac{p_2}{\rho}+\frac{V_2^2}{2}+gz_2=\frac{p_1}{\rho}+\frac{V_1^2}{2}+gz_1$ $\int_2^2\frac{dp}{\rho}+\frac{V_2^2}{2}+gz_2=\frac{V_1^2}{2}+gz_1$ $\frac{p}{\rho^k}=\text{constant}$ $\int_1^2\frac{dp}{\rho}=\frac{k}{k-1}\left(\frac{p_2}{\rho_2}-\frac{p_1}{\rho_1}\right)$ $\frac{k}{k-1}\frac{p_2}{\rho_2}+\frac{V_2^2}{2}+gz_2=\frac{k}{k-1}\frac{p_1}{\rho_1}+\frac{V_1^2}{2}+gz_1$	[four operations] [exponent] [derivative] → **Post-Secondary** [partial derivative] → **Post-Secondary** [integration] → **Post-Secondary** **Note: The main formula** $\frac{k}{k-1}\frac{p_2}{\rho_2}+\frac{V_2^2}{2}+gz_2$ $=\frac{k}{k-1}\frac{p_1}{\rho_1}+\frac{V_1^2}{2}+gz_1$ **is not based on calculus. The calculus-based formulas could be removed.**	[velocity] [density] [pressure] → **To be taught** [gravity]
5.5 Chapter Summary and Study Guide N/A	N/A	N/A
Chapter 8 Viscous Flow in Pipes		
8.1 General Characteristics of Pipe Flow **8.1.1 Laminar of Turbulent Flow** $\text{Re}=\frac{\rho VD}{\mu}$ **8.1.2 Entrance Region and Fully Developed Flow** $\frac{\ell_e}{D}=0.06\,\text{Re}$ (for turbulent flow) $\frac{\ell_e}{D}=4.4(\text{Re})^{1/6}$ (for turbulent flow) $10^4<\text{Re}<10^5$	[four operations] [coordinate system] [exponent]	[mass] [density] [force] [pressure] → **To be taught** [velocity] [momentum]
8.2.3 From Dimensional Analysis $\Delta p=F(V,\ell,D,\mu)$ $\frac{D\ \Delta p}{\mu V}=\phi\left(\frac{\ell}{D}\right)$ $\phi\left(\frac{\ell}{D}\right)=\frac{C\ell}{D}$ $C=\text{constant}$ $\frac{D\ \Delta p}{\mu V}=\frac{C\ell}{D}$ $\frac{\Delta p}{\ell}=\frac{C\mu V}{D^2}$ $Q=AV=\frac{(\pi/4C)\Delta pD^4}{\mu\ell}$	[four operations] [exponent]	[force] [velocity] [pressure] → **To be taught** [density] **Note: Special topics from 7.1 (Dimensional Analysis) need to be taught**

Table 2. (Continued).

Engineering Analytic Topics & Formulas **Subject: Fluid Mechanics**	**Math & Science Pre-requisite Topics & Skills**	
	[Math]	**[Physics]/{Chemistry}**
Chapter 8 Viscous Flow in Pipes (Continued)		
8.2.3 From Dimensional Analysis (Continued) $\Delta p = \frac{32\mu\ell V}{D^2}$ $\frac{\Delta p}{\frac{1}{2}\rho V^2} = \frac{(32\mu\ell V/D^2)}{\frac{1}{2}\rho V^2} = 64\left(\frac{\mu}{\rho VD}\right)\left(\frac{\ell}{D}\right) = \frac{64}{\mathrm{Re}}\left(\frac{\ell}{D}\right)$ $\Delta p = f\frac{\ell}{D}\frac{\rho V^2}{2}$ $f = \Delta p\left(\frac{D}{\ell}\right)\left(\frac{\rho V^2}{2}\right)$ $f = \frac{64}{\mathrm{Re}}$ $f = \frac{8\tau_w}{\rho V^2}$	↑ Refer to the previous page.	↑ Refer to the previous page.
8.2.4 Energy Considerations $\frac{p_1}{\gamma} + \alpha_1\frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \alpha_2\frac{V_2^2}{2g} + z_2 + h_L$ $\left(\frac{p_1}{\gamma} + z_1\right) - \left(\frac{p_2}{\gamma} + z_1\right) = h_L$ $p_1 = p_2 + \Delta p$ $z_2 - z_1 = \ell\sin\theta$ $h_L = \frac{2\tau\ell}{\gamma r}$ $h_L = \frac{4\ell\tau_w}{\gamma D}$	[four operations] [trigonometric functions]	[pressure] → **To be taught** [gravity]
8.3.5 Chaos and Turbulence N/A	N/A	N/A
8.4 Dimensional Analysis of Pipe Flow $h_L = h_{\mathrm{L\,major}} + h_{\mathrm{L\,minor}}$	[four operations]	**Note: Special topics from 7.1 (Dimensional Analysis) need to be taught**
8.4.1 Major Losses $h_L = h_{\mathrm{L\,major}} + h_{\mathrm{L\,minor}}$ $\Delta p = F(V, D, \ell, \varepsilon, \mu, \rho)$ $\frac{\Delta p}{\frac{1}{2}\rho V^2} = \tilde{\phi}\left(\frac{\rho VD}{\mu}, \frac{\ell}{D}, \frac{\varepsilon}{D}\right)$ $\mathrm{Re} = \frac{\rho VD}{\mu}$ $\frac{\Delta p}{\frac{1}{2}\rho V^2} = \frac{\ell}{D}\phi\left(\mathrm{Re}, \frac{\varepsilon}{D}\right)$ $f = \phi\left(\mathrm{Re}, \frac{\varepsilon}{D}\right)$ $\frac{p_1}{\gamma} + \alpha_1\frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \alpha_2\frac{V_2^2}{2g} + z_2 + h_L$ $h_{\mathrm{L\,major}} = f\frac{\ell}{D}\frac{V^2}{2g}$ $p_1 - p_2 = \gamma(z_2 - z_1) + \gamma h_L = \gamma(z_2 - z_1) + f\frac{\ell}{D}\frac{\rho V^2}{2}$ $\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$	[four operations] [areas of geometric shapes: circle, triangle] [logarithmic functions] **(To be taught or reviewed as a special skill)** [exponent] [square root] [graph]	[velocity] [pressure] → **To be taught** [force] [gravity] [density] **Note: Special topics from 7.1 (Dimensional Analysis) need to be taught**
8.4.2 Minor Losses $K_L = \frac{h_{\mathrm{L\,minor}}}{V^2/2g} = \frac{\Delta p}{\frac{1}{2}\rho V^2}$ $\Delta p = K_L\frac{1}{2}\rho V^2$ $h_{\mathrm{L\,minor}} = K_L\frac{V^2}{2g}$ $K_L = \phi(geometry, \mathrm{Re})$ $h_{\mathrm{L\,minor}} = K_L\frac{V^2}{2g} = f\frac{\ell_{eq}}{D}\frac{V^2}{2g}$ $\ell_{eq} = \frac{K_L D}{f}$ $A_1V_1 = A_3V_3$ $p_1A_3 - p_3A_3 = \rho A_3V_3(V_3 - V_1)$ $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + h_L$ $K_L = \frac{h_L}{V_1^2/2g}$ $K_L = \left(1 - \frac{A_1}{A_2}\right)^2$ $C_p = (p_2 - p_1)\left(\frac{\rho V_1^2}{2}\right)$		

Table 2. (Continued).

Engineering Analytic Topics & Formulas Subject: Fluid Mechanics	Math & Science Pre-requisite Topics & Skills [Math]	 [Physics]/{Chemistry}
Chapter 8 Viscous Flow in Pipes (Continued)		
8.4.3 Noncircular Conduits $f = \frac{C}{\mathrm{Re}_h}$ $\mathrm{Re}_h = \frac{\rho V D_h}{\mu}$ $D_h = \frac{4A}{P} = \frac{4(\pi D^2/4)}{\pi D} = D$ $h_L = f\frac{(\ell/D_h)V^2}{2g}$ **8.5 Pipe Flow Examples** N/A	[four operations] [exponent] [areas of geometric shapes]	[velocity] [gravity] [density]
8.5.1 Single Pipes N/A **8.5.2 Multiple Pipe Systems** N/A	[four operations] [exponent] [areas of geometric shapes]	[velocity] [gravity] [density]
8.6 Pipe Flowrate Measurement **8.6.1 Pipe Flowrate Meters** $Q_{ideal} = A_2V_2 = A_2\sqrt{\frac{2(p_1 - p_2)}{\rho(1-\beta^4)}}$ $Q = A_1V_1 = A_2V_2$ $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + h_L$ $Q = C_0Q_{ideal} = C_0A_0\sqrt{\frac{2(p_1 - p_2)}{\rho(1-\beta^4)}}$ $\beta = \frac{d}{D}$ $\mathrm{Re} = \frac{\rho VD}{\mu}$ $V = \frac{Q}{A_1}$ $Q = C_nQ_{ideal} = C_nA_n\sqrt{\frac{2(p_1 - p_2)}{\rho(1-\beta^4)}}$ $Q = C_vQ_{ideal} = C_vA_T\sqrt{\frac{2(p_1 - p_2)}{\rho(1-\beta^4)}}$ **8.6.2 Volume Flow Meters** N/A	[four operations] [exponent] [square root] [areas of geometric shapes]	[velocity] [pressure] → **To be taught** [gravity] [density]
8.7 Chapter Summary and Study Guide N/A	N/A	N/A
Chapter 9 Flow over Immersed Bodies		
9.1 General External Flow Characteristics **9.1.1 Lift and Drag Concepts** $\left.\begin{array}{l} dF_x = (p\ dA)\cos\theta + (\tau_w\ dA)\sin\theta \\ dF_y = -(p\ dA)\sin\theta + (\tau_w\ dA)\cos\theta \end{array}\right\} \rightarrow$ $\vec{D} = \int dF_x = \int p\ \cos\theta\ dA + \int \tau_w\ \sin\theta\ dA$ $\vec{L} = \int dF_y = -\int p\ \sin\theta\ dA + \int \tau_w\ \cos\theta\ dA$ $C_L = \frac{\vec{L}}{\frac{1}{2}\rho U^2 A}$ $C_D = \frac{\vec{D}}{\frac{1}{2}\rho U^2 A}$	[four operations] [areas of geometric shapes] [trigonometric functions] [integration] → **Post-Secondary** [derivative] → **Post-Secondary** **Note: The main formulas** $C_L = \frac{\vec{L}}{\frac{1}{2}\rho U^2 A}$ $C_D = \frac{\vec{D}}{\frac{1}{2}\rho U^2 A}$ **are not based on calculus. The calculus-based formulas could be removed.**	[force]

Table 2. (Continued).

Engineering Analytic Topics & Formulas **Subject: Fluid Mechanics**	**Math & Science Pre-requisite Topics & Skills**	
	[Math]	**[Physics]/{Chemistry}**
Chapter 9 Flow over Immersed Bodies Continued)		
9.1.2 Characteristics of Flow Past an Object N/A	N/A	[force] [Reynolds Number] → **To be taught as special topic**
9.3 Drag $C_D = \dfrac{\vec{D}}{\frac{1}{2}\rho U^2 A} \quad C_D = \phi(shape, \mathrm{Re}, Ma, Fr, \varepsilon/\ell)$	[four operations] [areas of geometric shapes]	[force] [density]
9.3.1 Friction Drag $\vec{D}_f = \frac{1}{2}\rho U^2 b \ell C_{Df}$		
9.3.2 Pressure Drag $D_p = \int \rho \cos\theta \, dA$ $C_{Dp} = \dfrac{\vec{D}_p}{\frac{1}{2}\rho U^2 A} = \dfrac{\int \rho \cos\theta \, dA}{\frac{1}{2}\rho U^2 A} = \dfrac{\int C_p \cos\theta \, dA}{A}$ $\vec{D} = f(U, \ell, \mu) \quad \vec{D} = C\mu\ell U \quad C_D = \dfrac{\vec{D}}{\frac{1}{2}\rho U^2 \ell^2} = \dfrac{2C\mu\ell U}{\rho U^2 \ell^2} = \dfrac{2C}{\mathrm{Re}}$	[four operations] [areas of geometric shapes] [integration] → **Post-Secondary** **The calculus-based formulas could be removed.**	[force] [density]
9.3.3 Drag Coefficient Data and Examples	[four operations] [areas of geometric shapes]	
9.4 Lift **9.4.1 Surface Pressure Distribution** $C_L = \dfrac{\vec{L}}{\frac{1}{2}\rho U^2 A} \quad C_L = \phi(shape, \mathrm{Re}, Ma, Fr, \varepsilon/\ell)$		
9.4.2 Circulation N/A		
9.5 Chapter Summary and Study Guide N/A	N/A	N/A
Chapter 10 Open Channel Flow		
10.1 General Characteristics of Open-Channel Flow $\mathrm{Re} = \rho V R_h / \mu \quad Fr = V/(g\ell)^{1/2}$	[four operations] [exponent] [trigonometric functions] [ellipse] → **To be taught or reviewed as a special topic**	[velocity] [gravity]
10.2 Surface Waves **10.2.1 Wave Speed** $\left.\begin{array}{l} -cy\not{b} = (-c+\delta V)(y+\delta y)\not{b} \\ c = \dfrac{(y+\delta y)\delta V}{\delta y} \\ \delta y << y \quad \rightarrow \quad c = y\dfrac{\delta V}{\delta y} \end{array}\right\} \rightarrow$ $\frac{1}{2}\gamma y^2 b - \frac{1}{2}\gamma(y+\delta y)^2 b = \rho b c y[(c-\delta V)-c]$ $F_1 = \dfrac{\gamma y_{c1} A_1 = \gamma(y+\delta y)^2 b}{2} \quad F_2 = \dfrac{\gamma y_{c2} A_2 = \gamma(y+\delta y)^2 b}{2}$	[four operations] [square root] [trigonometric functions] [derivative] → **Post-Secondary** **The calculus-based formulas could be removed.**	[velocity] [speed] [gravity]

Table 2. (Continued).

Engineering Analytic Topics & Formulas Subject: Fluid Mechanics	**Math & Science Pre-requisite Topics & Skills [Math]**	**[Physics]/{Chemistry}**
Chapter 10 Open Channel Flow (Continued)		
10.2 Surface Waves **10.2.1 Wave Speed** $\frac{\delta V}{\delta y}=\frac{g}{c} \quad c=\sqrt{gy} \quad \left.\begin{array}{l}\frac{V^2}{2g}+y=\text{constant}\\ \frac{V\,\delta V}{g}+\delta y=0\\ y\,\delta V+V\,\delta y=0\\ \frac{\delta y}{y}<<1 \rightarrow c\approx\sqrt{gy}\left(1+\frac{\delta y}{y}\right)^{1/2}\end{array}\right\}\rightarrow$	↑ Refer to the previous page.	↑ Refer to the previous page.
10.2.1 Wave Speed (Continued) $c=\left[\frac{g\lambda}{2\pi}\tanh\left(\frac{2\pi y}{\lambda}\right)\right]^{1/2} \quad y>>\lambda \rightarrow c=\sqrt{\frac{g\lambda}{2\pi}}$ $\tanh\left(\frac{2\pi y}{\lambda}\right)\rightarrow 1 \quad as \quad \frac{y}{\lambda}\rightarrow\infty$ $\tanh\left(\frac{2\pi y}{\lambda}\right)\rightarrow\frac{2\pi y}{\lambda} \quad as \quad \frac{y}{\lambda}\rightarrow 0$	[four operations] [square root] [trigonometric functions] [analytic geometry: hyperbolic tangent] **Post-secondary → To be taught** [derivative] **→ Post-Secondary** **The calculus-based formulas could be removed.**	[velocity] [speed] [gravity]
10.2.2 Froude Number Effects N/A	N/A	[velocity] [speed]
10.3 Energy Considerations $\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+h_L \quad \left.\begin{array}{l}z_1-z_2=S_0\ell\\ \frac{p_1}{\gamma}=y_1\\ \frac{p_2}{\gamma}=y_2\end{array}\right\}\rightarrow$ $y_1+\frac{V_1^2}{2g}+S_0\ell=y_2+\frac{V_2^2}{2g}+h_L$ $S_f=\frac{h_L}{\ell} \rightarrow y_1-y_2=\frac{\left(V_2^2-V_1^2\right)}{2g}+\left(S_f-S_0\right)\ell$ $\left.\begin{array}{l}S_f=0\\ S_0=0\end{array}\right\} \rightarrow y_1-y_2=\frac{\left(V_2^2-V_1^2\right)}{2g}$	[four operations] [exponent]	[velocity] [gravity] [potential energy]
10.3.1 Specific Energy $E=y+\frac{V^2}{2g} \quad E_1=E_2+\left(S_f-S_0\right)\ell \quad E=y+\frac{q^2}{2gy^2}$ $\frac{dE}{dy}=1-\frac{q^2}{gy^3}=0 \quad y_c=\left(\frac{q^2}{g}\right)^{1/3} \quad E_{min}=\frac{3y_c}{2}$ $V_c=\frac{q}{y_c}=\frac{\left(y_c^{3/2}g^{1/2}\right)}{y_c}=\sqrt{gy_c} \quad Fr\equiv V_c/\left(gy_c\right)^{1/2}=1$	[four operations] [derivative] **→ Post-Secondary** **The calculus-based formulas could be removed.**	[energy] [gravity] [velocity]
10.4 Uniform Depth Channel Flow **10.4.1 Uniform Flow Approximations** N/A	[areas of geometric shapes] [perimeter]	[velocity] [stress] **→ To be taught**

Table 2. (Continued).

Engineering Analytic Topics & Formulas Subject: Fluid Mechanics	**Math & Science Pre-requisite Topics & Skills**	
	[Math]	**[Physics]/{Chemistry}**
Chapter 10 Open Channel Flow (Continued)		
10.4.2 The Chezy and Manning Equations $\sum F_x = \rho Q(V_2 - V_1) = 0 \quad F_1 - F_2 - \tau_w P\ell + \vec{W}\sin\theta = 0$ $\sum F_x = 0$ $\left.\begin{array}{l} \tau_w = \frac{\vec{W}\sin\theta}{P\ell} = \frac{\vec{W}S_0}{P\ell} \\ \sin\theta \approx \tan\theta = S_0 \\ S_0 << 1 \\ \vec{W} = \gamma A\ell \\ R_h = A/P \end{array}\right\} \rightarrow \begin{array}{l} \tau_w = \frac{\gamma A\ell S_0}{P\ell} = \gamma R_h S_0 \\ \tau_w == K\rho\frac{V^2}{2} \quad K\rho\frac{V^2}{2} = \gamma R_h S_0 \\ V = C\sqrt{R_h S_0} \\ V = \frac{R_h^{2/3} S_0^{1/2}}{n} \end{array}$ $\rightarrow$ $V = \frac{\kappa}{n} A R_h^{2/3} S_0^{1/2} \quad Q = \frac{\kappa}{n} A R_h^{2/3} S_0^{1/2}$	[four operations] [exponent] [areas of geometric shapes] [trigonometric functions]	[pressure] → **To be taught** [velocity]
10.4.3 Uniform Depth Examples N/A	[four operations] [exponent] [areas of geometric shapes] [trigonometric functions]	[pressure] → **To be taught** [velocity]
10.5 Gradually Varied Flow N/A		
10.5.1 Classification of Surface Shapes N/A		
10.5.2 Examples of Gradually Varied Flows N/A		
10.6 Rapidly Varied Flow N/A	[four operations] [exponent] [square root]	[force] [velocity] [gravity] [pressure] → **To be taught**
10.6.1 The Hydraulic Jump $F_1 - F_2 = \rho Q(V_2 - V_1) = \rho V_1 y_1 b(V_2 - V_1)$ $\left.\begin{array}{l} F_1 = p_{c1}A_1 = \frac{\gamma y_1^2 b}{2} \quad p_{c1} = \frac{\gamma y_1}{2} \\ F_2 = p_{c2}A_2 = \frac{\gamma y_2^2 b}{2} \quad p_{c1} = \frac{\gamma y_2}{2} \end{array}\right\} \rightarrow$ $\frac{y_1^2}{2} - \frac{y_2^2}{2} = \frac{V_1 y_1}{g}(V_2 - V_1)$ $y_1 b_1 V_1 = y_2 b_2 V_2 = Q \quad y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + h_L$ $\frac{y_1^2}{2} - \frac{y_2^2}{2} = \frac{V_1 y_1}{g}\left(\frac{V_1 y_1}{y_2} - V_1\right) = \frac{V_1^2 y_1}{g y_2}(y_1 - y_2)$ $\left(\frac{y_2}{y_1}\right)^2 + \left(\frac{y_2}{y_1}\right) - 2Fr_1^2 = 0 \quad \left.\begin{array}{l} Fr_1 = \frac{V_1}{\sqrt{g y_1}} \\ \frac{y_2}{y_1} = \frac{1}{2}\left(-1 \pm \sqrt{1 + 8Fr_1^2}\right) \end{array}\right\} \rightarrow$ $\frac{y_2}{y_1} = \frac{1}{2}\left(-1 + \sqrt{1 + 8Fr_1^2}\right) \quad \frac{h_L}{y_1} = 1 - \frac{y_2}{y_1} + \frac{Fr_1^2}{2}\left[1 - \left(\frac{y_1}{y_2}\right)^2\right]$		

Table 2. (Continued).

Engineering Analytic Topics & Formulas Subject: Fluid Mechanics	Math & Science Pre-requisite Topics & Skills [Math]	 [Physics]/{Chemistry}
Chapter 10 Open Channel Flow (Continued)		
10.6.2 Sharp-Crested Weirs $\frac{p_A}{\gamma}+\frac{V_1^2}{2g}+z_A=(H+P_w-h)+\frac{u_2^2}{2g} \quad u_2=\sqrt{2g\left(h+\frac{V_1^2}{2g}\right)}$ $Q=\int_{(2)}u_2\ dA=\int_{h=0}^{h=H}u_2\ell\ dh$ $\ell=b \rightarrow Q=\sqrt{2g}b\int_0^H\left(h+\frac{V_1^2}{2g}\right)^{1/2}dh$ $Q=\frac{2}{3}\sqrt{2g}b\left[\left(H+\frac{V_1^2}{2g}\right)^{3/2}-\left(\frac{V_1^2}{2g}\right)^{3/2}\right] \left.\begin{matrix} P_w>>H \\ \frac{V_1^2}{2g}<<H\end{matrix}\right\}\rightarrow$ $Q=\frac{2}{3}\sqrt{2g}H^{3/2} \quad Q=C_{wr}\frac{2}{3}\sqrt{2g}bH^{3/2} \quad C_{wr}=0.611+0.075\left(\frac{H}{P_w}\right)$ $\ell=2(H-h)\tan\left(\frac{\theta}{2}\right) \quad \frac{V_1^2}{2g}<<H \rightarrow Q=\frac{8}{15}\tan\left(\frac{\theta}{2}\right)\sqrt{2g}H^{5/2}$ $Q=C_{wt}\frac{8}{15}\tan\left(\frac{\theta}{2}\right)\sqrt{2g}H^{5/2}$	[four operations] [exponent] [square root] [trigonometric functions] [integration] → **Post-Secondary** **Note: The main formulas are NOT based on calculus.** **The calculus-based formulas could be removed.**	[velocity] [gravity]
10.6.3 Broad-Crested Weirs $H+P_w+\frac{V_1^2}{2g}=y_c+p_w+\frac{V_c^2}{2g} \quad H-y_c=\frac{(V_c^2-V_1^2)}{2g}=\frac{V_c^2}{2g}$ $\left.\begin{matrix} V_2=V_c=(gy_c)^{1/2} \\ V_c^2=gy_c\end{matrix}\right\} \rightarrow H-y_c=\frac{y_c}{2} \rightarrow y_c=\frac{2H}{3}$	[four operations] [exponent] [square root]	[velocity] [gravity]
10.6.3 Broad-Crested Weirs (Continued) $Q=by_2V_2=by_cV_c=by_c(gy_c)^{1/2}=b\sqrt{g}y_c^{3/2} \rightarrow$ $Q=b\sqrt{g}\left(\frac{2}{3}\right)^{3/2}H^{3/2} \quad Q=C_{wb}b\sqrt{g}\left(\frac{2}{3}\right)^{3/2}H^{3/2}$ $C_{wb}=\frac{0.65}{(1+H/P_w)^{1/2}}$		
10.6.4 Underflow Gates $q=C_da\sqrt{2gy_1}$		
10.7 Chapter Summary and Study Guide N/A	N/A	N/A
Chapter 11 Compressible Flow		
11.3 Categories of Compressible Flow $r=(t-t_{wave})c \quad \sin\alpha=\frac{c}{V}=\frac{1}{Ma}$	[four operations] [trigonometric functions]	[velocity] [speed of sound]
11.4.2 Converging-Diverging Duct Flow $\frac{p}{p^k}=\text{constant}=\frac{p_0}{p_0^k} \quad \frac{dp}{\rho}+d\left(\frac{V^2}{2}\right)=0$ $\frac{p_0^{1/k}}{\rho_0}\frac{dp}{(p)^{1/k}}+d\left(\frac{V^2}{2}\right)=0$	[four operations] [exponent] [square root]	[pressure] → **To be taught** [density] [velocity] {**Ideal Gas Law**} → **Post-secondary → to be taught**

Table 2. (Continued).

Engineering Analytic Topics & Formulas Subject: Fluid Mechanics	Math & Science Pre-requisite Topics & Skills [Math]	 [Physics]/{Chemistry}
Chapter 11 Compressible Flow (Continued)		
11.4.2 Converging-Diverging Duct Flown (Continued) $\frac{k}{k-1}\left(\frac{p_0}{\rho_0}-\frac{p}{\rho}\right)-\frac{V^2}{2}=0 \quad \frac{kR}{k-1}(T_0-T)-\frac{V^2}{2}=0$ $\left.\begin{array}{l} c_p(T_0-T)-\frac{V^2}{2}=0 \\ \check{h}_2-\check{h}_1=c_p(T_2-T_1) \end{array}\right\} \rightarrow \check{h}_0-\left(\check{h}+\frac{V^2}{2}\right)=0$	↑ Refer to the previous page.	↑ Refer to the previous page.
11.4.2 Converging-Diverging Duct Flow (Continued) $\frac{T}{T_0}=\frac{1}{1+[(k-1)/2]Ma^2} \quad \left(\frac{p}{p_0}\right)\left(\frac{\rho_0}{\rho}\right)=\frac{T}{T_0} \quad \left(\frac{p}{p_0}\right)=\left(\frac{T}{T_0}\right)^{k/(k-1)}$ $\left.\begin{array}{l} \frac{T}{T_0}=\frac{1}{1+[(k-1)/2]Ma^2} \\ \left(\frac{p}{p_0}\right)=\left(\frac{T}{T_0}\right)^{k/(k-1)} \end{array}\right\} \rightarrow \frac{p}{p_0}=\left\{\frac{1}{1+[(k-1)/2]Ma^2}\right\}^{k/(k-1)}$ $\left.\begin{array}{l} \frac{T}{T_0}=\frac{1}{1+[(k-1)/2]Ma^2} \\ \left(\frac{p}{p_0}\right)\left(\frac{\rho_0}{\rho}\right)=\frac{T}{T_0} \\ \frac{p}{p_0}=\left\{\frac{1}{1+[(k-1)/2]Ma^2}\right\}^{k/(k-1)} \end{array}\right\} \rightarrow$ $\frac{\rho_0}{\rho}=\left\{\frac{1}{1+[(k-1)/2]Ma^2}\right\}^{k/(k-1)}$ $\frac{p^*}{p_0}=\left(\frac{2}{k+1}\right)^{k/(k-1)} \quad \left(\frac{p^*}{p_0}\right)_{k=1.4}=0.528 \quad p^*_{k=1.4}=0.528p_{atm}$ $\frac{T^*}{T_0}=\frac{2}{k+1} \quad \left(\frac{T^*}{T_0}\right)_{k=1.4}=0.833 \quad T^*_{k=1.4}=0.833T_{atm}$	[four operations] [exponent] [square root]	[pressure] → **To be taught** [density] [velocity] {**Ideal Gas Law**} → **Post-secondary → to be taught**
11.4.2 Converging-Diverging Duct Flow (Continued) $\left.\begin{array}{l} Ma=1 \\ p=\rho RT \\ \frac{p^*}{p_0}=\left(\frac{2}{k+1}\right)^{k/(k-1)} \\ \frac{T^*}{T_0}=\frac{2}{k+1} \end{array}\right\} \rightarrow$ $\frac{\rho^*}{\rho_0}=\left(\frac{\rho^*}{T^*}\right)\left(\frac{T_0}{p_0}\right)=\left(\frac{2}{k+1}\right)^{k/(k-1)}\left(\frac{k+1}{2}\right)=\left(\frac{2}{k+1}\right)^{k/(k-1)}$ $\left(\frac{\rho^*}{\rho_0}\right)_{k=1.4}=0.634$ $\rho AV=\rho^*A^*V^* \quad \frac{A}{A^*}=\left(\frac{\rho^*}{\rho}\right)\left(\frac{V^*}{V}\right) \quad \begin{array}{l} V^*=\sqrt{RT^*k} \\ V=Ma\sqrt{RTk} \end{array}$ $\frac{A}{A^*}=\frac{1}{Ma}\left(\frac{\rho^*}{\rho_0}\right)\left(\frac{\rho_0}{\rho}\right)\sqrt{\frac{(T^*/T_0)}{T/T_0}}$	[four operations] [exponent] [square root]	[pressure] → **To be taught** [density] [velocity] {**Ideal Gas Law**} → **Post-secondary → to be taught**

Table 2. (Continued).

Engineering Analytic Topics & Formulas **Subject: Fluid Mechanics**	**Math & Science Pre-requisite Topics & Skills**	
	[Math]	**[Physics]/{Chemistry}**
Chapter 11 Compressible Flow (Continued)		
11.4.2 Converging-Diverging Duct Flow (Continued) $\frac{T}{T_0}=\frac{1}{1+[(k-1)/2]Ma^2}$ $\frac{p}{p_0}=\left\{\frac{1}{1+[(k-1)/2]Ma^2}\right\}^{k/(k-1)}$ $\frac{T^*}{T_0}=\frac{2}{k+1}$ $\frac{\rho^*}{\rho_0}=\left(\frac{\rho^*}{T^*}\right)\left(\frac{T_0}{p_0}\right)=\left(\frac{2}{k+1}\right)^{k/(k-1)}\left(\frac{k+1}{2}\right)=\left(\frac{2}{k+1}\right)^{k/(k-1)}$ $\frac{A}{A^*}=\frac{1}{Ma}\left(\frac{\rho^*}{\rho_0}\right)\left(\frac{\rho_0}{\rho}\right)\sqrt{\frac{(T^*/T_0)}{T/T_0}}$ } $\rightarrow$ $\frac{A}{A^*}=\frac{1}{Ma}\left\{\frac{1+[(k-1)/2]Ma^2}{1+[(k-1)/2]}\right\}^{(k+1)/[2(k-1)]}$	[four operations] [exponent] [square root]	[pressure] → **To be taught** [density] [velocity] {**Ideal Gas Law**} → **Post-secondary** → **to be taught**
11.4.3 Constant Area Duct Flow N/A	N/A	[density] [velocity] [pressure] [friction] → **To be taught** [acceleration]
11.5.3 Normal Shock Waves $\rho V=\text{constant}$ $p+\rho V^2=\text{constant}$ $p+\frac{(\rho V)^2RT}{p}=\text{constant}$ $\check{h}+\frac{V^2}{2}=\check{h}_0=\text{constant}$ $\check{h}-\check{h}_0=c_p(T-T_0)$ $p=\rho RT$ $T+\frac{(\rho V)^2T^2}{2c_p(p^2/R^2)}=T_0=\text{constant}$ $\frac{p_y}{p_x}=\left(\frac{p_y}{p_a}\right)\left(\frac{p_a}{p_x}\right)$ $\frac{p_y}{p_a}=\frac{1+k}{1+k\text{Ma}_y^2}$ $\frac{p_x}{p_a}=\frac{1+k}{1+k\text{Ma}_x^2}$ $\frac{p_y}{p_x}=\frac{1+k\text{Ma}_x^2}{1+k\text{Ma}_y^2}$ $\frac{p_y}{p_x}=\left(\frac{p_y}{p^*}\right)\left(\frac{p^*}{p_x}\right)$ $\frac{p}{p^*}=\frac{1}{\text{Ma}}\left\{\frac{(k+1)/2}{1+[(K-1)/2]\text{Ma}^2}\right\}^{1/2}$ $p_x+\rho_xV_x^2=p_y+\rho_yV_Y^2$ $\frac{\rho V^2}{p}=\frac{V^2}{RT}=\frac{kV^2}{RTk}=k\text{Ma}^2$ $\frac{T_y}{T_x}=\left(\frac{T_y}{T^*}\right)\left(\frac{T^*}{T_x}\right)$ $\frac{T_y}{T^*}=\frac{(k+1)/2}{1+[(k-1)/2]\text{Ma}_y^2}$ $\frac{T_x}{T^*}=\frac{(k+1)/2}{1+[(k-1)/2]\text{Ma}_x^2}$ } $\rightarrow$	[four operations] [exponent]	{**Ideal Gas Law**} → **Post-secondary** → **to be taught** [temperature] [density] [pressure] → **To be taught** [speed] [velocity] [graph]

Table 2. (Continued).

Engineering Analytic Topics & Formulas **Subject: Fluid Mechanics**	**Math & Science Pre-requisite Topics & Skills** **[Math]**	 **[Physics]/{Chemistry}**
Chapter 11 Compressible Flow (Continued)		
11.5.3 Normal Shock Waves (Continued) $\frac{T_y}{T_x} = \frac{1+[(k-1)/2]Ma_x^2}{1+[(k-1)/2]Ma_y^2}$ $\frac{p_y}{p_x} = \left(\frac{T_y}{T_x}\right)\left(\frac{\rho_y}{\rho_x}\right)$ $\rho_x V_x = \rho_y V_y$ $\frac{p_y}{p_x} = \left(\frac{T_y}{T_x}\right)\left(\frac{V_x}{V_y}\right)$ $\frac{p_y}{p_x} = \left(\frac{T_y}{T_x}\right)^{1/2}\left(\frac{Ma_x}{Ma_y}\right)$ $\frac{p_y}{p_x} = \left\{\frac{1+[(k-1)/2]Ma_x^2}{1+[(k-1)/2]Ma_y^2}\right\}^{1/2}\frac{Ma_x}{Ma_y}$ $Ma_y^2 = \frac{Ma_x^2+[2/(k-1)]}{[2k/(k-1)]Ma_x^2-1}$ $\frac{p_y}{p_x} = \frac{2k}{k+1}Ma_x^2 - \frac{k-1}{k+1}$ $\frac{T_y}{T_x} = \frac{\{1+[(k-1)/2]Ma_x^2\}\{2k/(k-1)\}Ma_x^2-1}{\{(k+1)^2/2(k-1)\}Ma_x^2}$ $\frac{\rho_y}{\rho_x} = \frac{V_x}{V_y}$ $\frac{\rho_y}{\rho_x} = \left(\frac{p_y}{p_x}\right)\left(\frac{T_x}{T_y}\right)$ $\frac{\rho_y}{\rho_x} = \frac{V_x}{V_y} = \frac{(k+1)Ma_x^2}{(k-1)Ma_x^2+2}$ $\frac{p_{0,y}}{p_{0,x}} = \left(\frac{p_{0,y}}{p_y}\right)\left(\frac{p_y}{p_x}\right)\left(\frac{p_x}{p_{0,x}}\right)$ $\frac{p_{0,y}}{p_{0,x}} = \frac{\left(\frac{k+1}{2}Ma_x^2\right)^{k/(k-1)}\left(1+\frac{k-1}{2}Ma_x^2\right)^{k/(1-k)}}{\left(\frac{2k}{k+1}Ma_x^2-\frac{k-1}{k+1}\right)^{1/(k-1)}}$	↑ Refer to the previous page.	↑ Refer to the previous page.
11.6 Analogy between Compressible and Open-Channel Flows $Ma = \frac{V}{c}$ $Fr = \frac{V_{oc}}{\sqrt{gy}}$ $c_{oc} = \sqrt{gy}$ $Fr = \frac{V_{oc}}{c_{oc}}$ $\rho AV = \text{constant}$ $ybV_{oc} = \text{constant}$ $c = \sqrt{(\text{constant})k\rho^{k-1}}$	[four operations] [square root] [areas of geometric shapes]	[density] [velocity] [gravity] [mass]
11.7 Two-Dimensional Compressible Flow $V_{t1} = V_{t2}$	[four operations] [triangle]	[velocity]
11.8 Chapter Summary and Study Guide N/A	N/A	N/A
Chapter 12 Turbomachines		
12.1 Introduction N/A	N/A	[force] [work] [energy] [power]
12.2 Basic Energy Considerations $\vec{V} = \vec{W} + \vec{U}$ $U = \omega r$	[four operations] [radius]	[velocity]

Table 2. (Continued).

Engineering Analytic Topics & Formulas Subject: Fluid Mechanics	Math & Science Pre-requisite Topics & Skills [Math]	 [Physics]/{Chemistry}
Chapter 12 Turbomachines (Continued)		
12.3 Basic Angular Momentum Considerations $\sum(\vec{r}\times\vec{F})=\int_{cs}(\vec{r}\times\vec{V})\rho\vec{V}\cdot\hat{n}\;dA$ $T_{shaft}=-\dot{m}_1(r_1V_{\theta 1})+-\dot{m}_2(r_1V_{\theta 2})$ $m=\rho Q \quad \dot{W}_{shaft}=T_{shaft}\omega \quad \dot{W}_{shaft}=-\dot{m}_1(U_1V_{\theta 1})+-\dot{m}_2(U_1V_{\theta 2})$ $w_{shaft}=\frac{\dot{W}_{shaft}}{\dot{m}} \quad w_{shaft}=-U_1V_{\theta 1}+U_1V_{\theta 2} \quad V^2=V_\theta^2+V_x^2$ $V_x^2+(V_\theta-U)^2=W^2 \quad V_\theta U=\frac{V^2+U^2-W^2}{2}$ $w_{shaft}=\frac{V_2^2-V_1^2+U_2^2-U_1^2-(W_2^2-W_1^2)}{2}$	[sigma notation] [integration] → **Post-Secondary** [special math: cross product] → **To be taught as a special math topic** [analytic geometry] → **12th (To be taught as a special math topic) or Post-Secondary** [areas of geometric shapes]	[density] [torque] → **Post-secondary** → **To be taught** [momentum]
12.4 The Centrifugal Pump N/A	[four operations] [triangle] [trigonometric functions] [areas of geometric shapes]	[velocity] [density] [gravity]
12.4.1 Theoretical Considerations $\vec{V}_1=\vec{W}_1+\vec{U}_1 \quad U_1=r_1\omega$ $\vec{V}_2=\vec{W}_2+\vec{U}_2 \quad U_2=r_2\omega \quad \dot{m}_1=\dot{m}_2=\dot{m}$ $T_{shaft}=\dot{m}(r_2V_{\theta 2}-r_1V_{\theta 1}) \quad T_{shaft}=\rho Q(r_2V_{\theta 2}-r_1V_{\theta 1})$ $\dot{W}_{shaft}=T_{shaft}\omega \quad \dot{W}_{shaft}=\rho Q\omega(r_2V_{\theta 2}-r_1V_{\theta 1})$ $\dot{W}_{shaft}=\rho Q(U_2V_{\theta 2}-U_1V_{\theta 1}) \quad w_{shaft}=\frac{\dot{W}_{shaft}}{\rho Q}=U_2V_{\theta 2}-U_1V_{\theta 1}$ $\dot{W}_{shaft}=\rho gQh_i \quad h_i=\frac{1}{g}(U_2V_{\theta 2}-U_1V_{\theta 1})$		
12.4.1 Theoretical Considerations (Continued) $h_i=\frac{1}{2g}\left[(V_2^2-V_1^2)+(U_2^2-U_1^2)+(W_2^2-W_1^2)\right] \quad h_i=\frac{U_2V_{\theta 2}}{g}$ $\cot\beta_2=\frac{U_2-V_{\theta 2}}{V_{r2}} \quad h_i=\frac{U_2^2}{g}-\frac{U_2V_{r2}\cot\beta_2}{g} \quad Q=2\pi r_2b_2V_{r2}$ $h_i=\frac{U_2^2}{g}-\frac{U_2\cot\beta_2}{2\pi r_2b_2g}Q$	[four operations] [triangle] [trigonometric functions] [areas of geometric shapes]	
12.4.2 Pump Performance Characteristics $h_a=\frac{p_2-p_1}{\gamma}+z_2-z_1+\frac{V_2^2-V_1^2}{2g}$ $h_a=h_p=h_s-h_L \quad h_a\approx\frac{p_2-p_1}{\gamma} \quad \wp_f=\gamma Qh_a$ $\wp_f=\text{water horsepower}=\frac{\gamma Qh_a}{550}$ $\eta=\frac{\text{power gained by thevfluid}}{\text{shaft power driving the pump}}=\frac{\wp_f}{\dot{W}_{shaft}}$ $\eta=\frac{\gamma Qh_a/550}{bhp} \quad \eta=\eta_h\eta_m\eta_v$	[four operations] [areas of geometric shapes] [unit conversion]	[pressure] → **To be taught** [velocity] [gravity]

Table 2. (Continued).

Engineering Analytic Topics & Formulas **Subject: Fluid Mechanics**	**Math & Science Pre-requisite Topics & Skills**	
	[Math]	**[Physics]/{Chemistry}**
Chapter 12 Turbomachines (Continued)		
12.4.3 Net Positive Suction Head (NPSH) $NPSH = \frac{p_s}{\gamma} + \frac{V_s^2}{2g} - \frac{p_v}{\gamma} \quad \frac{p_{atm}}{\gamma} - z_1 = \frac{p_s}{\gamma} + \frac{V_s^2}{2g} = \sum h_L \quad \rightarrow$ $\frac{p_s}{\gamma} + \frac{V_s^2}{2g} = \frac{p_{atm}}{\gamma} - z_1 - \sum h_L \quad \rightarrow$ $NPSH = \frac{p_{atm}}{\gamma} - z_1 - \sum h_L - \frac{p_v}{\gamma}$	[four operations] [sigma notation]	[pressure] → **To be taught** [velocity] [gravity] [density]
12.4 4 System Characteristics and Pump Selection $h_p = z_2 - z_1 + \sum h_L \quad h_p = z_2 - z_1 + KQ^2$		[velocity] [density]
12.5 Dimensionless Parameters and Similarity Laws $\text{dependent variable} = f(D, \ell_i, \varepsilon, Q, \omega, \mu, \rho)$ $\text{dependent pi term} = \phi\left(\frac{\ell_i}{D}, \frac{\varepsilon}{D}, \frac{Q}{\omega D^3}, \frac{\rho \omega D^2}{\mu}\right)$ $C_H = \frac{g h_a}{\omega^2 D^2} = \phi_1\left(\frac{\ell_i}{D}, \frac{\varepsilon}{D}, \frac{Q}{\omega D^3}, \frac{\rho \omega D^2}{\mu}\right)$ $C_\wp = \frac{\dot{W}_{shaft}}{\rho \omega^3 D^5} = \phi_2\left(\frac{\ell_i}{D}, \frac{\varepsilon}{D}, \frac{Q}{\omega D^3}, \frac{\rho \omega D^2}{\mu}\right)$ $\eta = \frac{\rho g Q h_a}{\dot{W}_{shaft}} = \phi_3\left(\frac{\ell_i}{D}, \frac{\varepsilon}{D}, \frac{Q}{\omega D^3}, \frac{\rho \omega D^2}{\mu}\right)$ $\frac{g h_a}{\omega^2 D^2} = \phi_1\left(\frac{Q}{\omega D^3}\right) \quad \frac{\dot{W}_{shaft}}{\rho \omega^3 D^5} = \phi_2\left(\frac{Q}{\omega D^3}\right) \quad \eta = \phi_3\left(\frac{Q}{\omega D^3}\right)$ $\left(\frac{Q}{\omega D^3}\right)_1 = \left(\frac{Q}{\omega D^3}\right)_2 \quad \left(\frac{g h_a}{\omega^2 D^2}\right)_1 = \left(\frac{g h_a}{\omega^2 D^2}\right)_2$ $\left(\frac{\dot{W}_{shaft}}{\rho \omega^3 D^5}\right)_1 = \left(\frac{\dot{W}_{shaft}}{\rho \omega^3 D^5}\right)_2 \quad \eta = \eta_2$	[four operations] [ratio]	[gravity] [density] [energy] [velocity]
12.5.1 Special Pump Scaling Laws $\frac{Q_1}{Q_2} = \frac{\omega_1}{\omega_2} \quad \frac{h_{a1}}{h_{a2}} = \frac{\omega_1^2}{\omega_2^2} \quad \frac{\dot{W}_{shaft1}}{\dot{W}_{shaft2}} = \frac{\omega_1^3}{\omega_2^3} \quad \frac{Q_1}{Q_2} = \frac{D_1^3}{D_2^3} \quad \frac{h_{a1}}{h_{a2}} = \frac{D_1^2}{D_2^2}$ $\frac{\dot{W}_{shaft1}}{\dot{W}_{shaft2}} = \frac{D_1^5}{D_2^5} \quad \frac{1-\eta_2}{1-\eta_1} \approx \left(\frac{D_1}{D_2}\right)^{1/5}$	[four operations] [areas of geometric shapes: circle, triangle] [exponent] [ratio]	[velocity] [power] [energy]
12.5.2 Specific Speed $\frac{(Q/\omega D^3)^{1/2}}{(g h_a/\omega^2 D^2)^{3/4}} = \frac{\omega \sqrt{Q}}{(g h_a)^{3/4}} = N_s \quad N_{sd} = \frac{\omega(rpm)\sqrt{Q(gpm)}}{[h_a(ft)]^{3/4}}$	[four operations] [ratio]	[speed]
12.5.3 Suction Specific Speed $S_s = \frac{\omega \sqrt{Q}}{[g(NPSH_R)]^{3/4}} \quad S_{sd} = \frac{\omega(rpm)\sqrt{Q(gpm)}}{[NPSH_R(ft)]^{3/4}}$		
12.6 Axial-Flow and Mixed-Flow Pump N/A	[graph]	
12.7 Fans $\left(\frac{p_a}{\rho \omega^2 D^2}\right)_1 = \left(\frac{p_a}{\rho \omega^2 D^2}\right)_2$	[four operations] [areas of geometric shapes: circle, triangle] [ratio]	[speed] [pressure] → **To be taught** [density]

Table 2. (Continued).

Engineering Analytic Topics & Formulas Subject: Fluid Mechanics	Math & Science Pre-requisite Topics & Skills [Math]	[Physics]/{Chemistry}
Chapter 12 Turbomachines (Continued)		
12.8.2 Reaction Turbines $C_Q = \frac{Q}{\omega D^3} \quad C_H = \frac{gh_T}{\omega^2 D^2} \quad C_\wp = \frac{\dot{W}_{shaft}}{\rho\omega^3 D^5} \quad \eta = \frac{\dot{W}_{shaft}}{\rho g Q h_T}$ $C_H = \phi_1(C_Q) \quad C_\wp = \phi_2(C_Q) \quad \eta = \phi_3(C_Q) \quad \eta = \frac{C_\wp}{C_H C_Q}$ $N'_s = \frac{\omega\sqrt{\dot{W}_{shaft}/\rho}}{(gh_T)^{5/4}} \quad N'_{sd} = \frac{\omega(rpm)\sqrt{\dot{W}_{shaft}(bhp)}}{[h_T(ft)]^{5/4}}$	[four operations] [square root] [exponent]	[power] [speed] [force] [density] [gravity]
12.9 Compressible Flow Turbomachines **12.9.1 Compressors** $\left(\frac{R\dot{m}\sqrt{kRT_{01}}}{D^2 p_{01}}\right)_{test} = \left(\frac{R\dot{m}\sqrt{kRT_{01}}}{D^2 p_{01}}\right)_{std}$ $\dot{m}_{std} = \frac{\dot{m}_{std}\sqrt{T_{01\ test}/T_{0\ std}}}{p`_{01\ test}/p_{0\ std}}$ $\frac{ND}{\sqrt{kRT_{01}}} \quad N_{std} = \frac{N}{\sqrt{T_{01}/T_{std}}}$	[four operations] [square root] [graph]	[mass] [pressure] [friction] → **To be taught** [velocity] [temperature]
12.9.2 Compressible Flow Turbines N/A	N/A	[mass] [pressure] → **To be taught** [friction] → **To be taught** [velocity] [temperature]
12.10 Chapter Summary and Study Guide N/A	N/A	N/A
THE END		

Table 3. Pre-Requisite Mathematics and Science Topics to Be Reviewed Before Teaching the Pre-Calculus Portions of Fluid Mechanics Topics to 9^{th} Grade Students

Pre-Requisites to be Taught or Reviewed	
[Math]	**[Physics]/{Chemistry}**
1. [analytic geometry] → **12^{th} (To be taught as a special skill)**	1. {**absolute temperature**} → **To be taught as a special topic**
2. [analytic geometry: hyperbolic tangent] **Post-secondary → To be taught as a special skill**	2. [acceleration]
3. [areas of geometric shapes: circle, triangle, etc.]	3. [density]
4. [cylinder]	4. [energy]
5. [derivative] → **12^{th} (To be taught as a special skill)**	5. [force]
6. cross product] → **To be taught as a special math topic**	6. [friction] → **To be taught as a special topic**
7. [ellipse] → **To be taught**	7. **{gas/liquid}** → **To be taught as a special topic**
8. [exponent]	8. [graph]
9. [four operations]	9. [gravity]
10. [graph]	10. [heat]
11. [height]	11. {**Ideal Gas Law**} → **To be taught as a special topic**
12. [integration] → **12^{th} (To be taught as a special skill)**	12. **{intermolecular cohesive force}** → **To be taught as a special topic**
13. [logarithmic functions] → **To be taught or reviewed as a special skill**	13. [mass]
14. [perimeter]	14. [molecule]
15. [Pythagorean Theorem]	15. [momentum]
16. [prism]	16. [Newton's 1^{st}, 2^{nd} and 3^{rd} Laws] → **To be taught or reviewed as a special topic**
17. [radius]	17. [potential energy]
18. [ratio]	18. [power]
19. [sigma notation]	19. [pressure] → **To be taught or reviewed as a special topic**
20. [square root]	20. [Reynolds Number] → **To be taught or reviewed as a special topic**
21. [triangle]	21. [speed]
22. [trigonometric functions]	22. [speed of sound] → **To be taught or reviewed as a special topic**
23. [unit conversion]	23. [stress] → **To be taught or reviewed as a special topic**
24. [volume]	24. [temperature]
	25. [torque] → **To be taught or reviewed as a special topic**
	26. [velocity]
	27. [weight]
	28. [work]

Table 4. Pre-Calculus Based Fluid Mechanics Topics That Possibly Could Be Taught at 9th Grade (Chapters and sections)

Chapter/Section	Page Numbers	Number of Pages
Chapter 1 – Introduction (pp. 1-30 → 30 pages sub-total. 10 sections out of 11)		
1.1 Some Characteristics of Fluid	1-13	13
1.2 Dimensions, Dimensional Homogeneity, and Units		
1.3 Analysis of Fluid Mechanics Behavior		
1.4 Measures of Fluid Mechanics Mass and Weight		
1.4.1 Density		
1.4 2 Specific Weight		
1.4.3 Specific Gravity		
1.5 Ideal Gas Law		
1.7 Compressibility of Fluids	20-30	11
1.7.1 Bulk Modulus		
1.7.2 Compression and Expansion of Gases		
1.7.3 Speed of Sound		
1.8 Vapor Pressure		
1.9 Surface Tension		
1.10 A Brief Look Back in History		
1.11 Chapter Summary and Study Guide		
Chapter 2 Fluid Statics (pp. 38-79 → 42 pages sub-total. 9 sections out of 13)		
2.3 Pressure Variation in a Fluid at Rest (Concept only)*		
2.3.1 Incompressible Fluid	42-56	15
2.3.2 Compressible Fluid		
2.4 Standard Atmosphere		
2.5 Measurement of Pressure		
2.6 Monometry		
2.6.1 Piezometer Tube		
2.6.2 U-Tube Manometer		
2.6.3 Inclined-Tube Manometer		
2.7 Mechanical and Electronic Pressure Measuring Devices		
2.9 Pressure Prism	63-72	10
2.10 Hydrostatic Force on a Curves Surface		
2.11 Buoyancy, Flotation, and Stability		
2.11.1 Archimedes' Principle		
2.11.2 Stability		
2.13 Chapter Summary and Study Guide	78-79	2
Chapter 3 Elementary Fluid Dynamics – The Bernoulli Equation (pp. 95-135 → 41 pages sub-total. 8 sections out of 9)		
3.1 Newton's Second Law	95-101	7
3.2 F = ma along a Streamline		
3.4 Physical Interpretation	104-135	32
3.5 Static, Stagnation, Dynamic, and Total Pressure		
3.6 Examples of Use of the Bernoulli Equation		
3.6.1 Free Jets		
3.6.2 Confined Flows		
3.6.3 Flowrate Measurement		
3.7 The Energy Line and the Hydraulic Grade Line		
3.8 Restrictions on Use of the Bernoulli Equation		
3.8.1 Compressibility Effects		
3.8.3 Rotational Effects		

* Basic principles covered under this section heading could be explored; but the formulas used are calculus-based.

List 1A. (Continued)

Chapter/Section	Page Numbers	Number of Pages
Chapter 3 Elementary Fluid Dynamics – The Bernoulli Equation (Continued)		
3.8.4 Other Restrictions	↑	↑
3.9 Chapter Summary and Study Guide		
Chapter 4 Fluid Kinematics (pp. 150-184 → 35 pages sub-total. 3 sections out of 5)		
4.3 Control Volume and System Representations	168-169	2
4.4 The Reynolds Transport Theorem	170-171	2
4.4.7 Selection of a Control Volume	182-182	3
4.5 Chapter Summary and study Guide		
Chapter 5 Finite Control Volume Analysis (pp. 192-252 → 61 pages sub-total 2 sections out of 5)		
5.1 Conservation of Mass – The Continuity Equation (Concept only)*		
5.1.2 Fixed, Non-deforming Control Volume	195-200	6
5.3.3 Comparison of the Energy Equation with the Bernoulli Equation	236-246	11
5.3.4 Application of the Energy Equation to Non-uniform Flow		
5.3.5 Combination of the Energy Equation and the Moment-of-momentum Equation		
5.4.4 Application of the Loss Form of the Energy Equation	249-252	4
5.5 Chapter Summary and Study Guide		
Chapter 6 Differential Analysis of Fluid Flow (pp. 272-334 → 63 pages sub-total. 0 sections out of 11)		
Chapter 7 Similitude, Dimensional Analysis, and Modeling (pp. 346-391 → 46 pages sub-total. 0 sections out of 11)		
Chapter 8 Viscous Flow in Pipes (pp. 401-472 → 72 pages sub-total. 5 sections out of 7)		
8.2 Fully Developed Laminar Flow (Concept only)*		
8.2.4 Energy Considerations	416-417	2
8.4 Dimensional Analysis of Pipe Flow	430-472	43
8.4.1 Major Losses		
8.4.2 Minor Losses		
8.4.3 Noncircular Conduits		
8.5 Pipe Flow Examples		
8.5.1 Single Pipes		
8.5.2 Multiple Pipe Systems		
8.6 Pipe Flowrate Measurement		
8.6.1 Pipe Flowrate Meters		
8.6.2 Volume Flow Meters		
8.7 Chapter Summary and Study Guide		
Chapter 9 Flow over Immersed Bodies (pp. 483-550 → 68 pages sub-total. 4 sections out of 5)		
9.1 General External Flow Characteristics	484-493	10
9.1.1 Lift and Drag Concepts		
9.1.2 Characteristics of Flow Past an Object		
9.3 Drag	518-550	33
9.3.1 Friction Drag		
9.3.2 Pressure Drag		
9.3.3 Drag Coefficient Data and Examples		
9.4 Lift		
9.4.1 Surface Pressure Distribution		
9.4.2 Circulation		
9.5 Chapter Summary and Study Guide		

List 1A. (Continued)

Chapter/Section	Page Numbers	Number of Pages
Chapter 10 Open Channel Flow (Whole Chapter; pp. 561-605 → 45 pages sub-total. 7 sections out of 7)		
10.1 General Characteristics of Open-Channel Flow	561-573	13
10.2 Surface Waves		
10.2.1 Wave Speed		
10.2.2 Froude Number Effects		
10.3 Energy Considerations		
10.3.1 Specific Energy		
10.4 Uniform Depth Channel Flow	574-605	32
10.4.1 Uniform Flow Approximations		
10.4.2 The Chezy and Manning Equations		
10.4.3 Uniform Depth Examples		
10.5 Gradually Varied Flow		
10.5.1 Classification of Surface Shapes		
10.5.2 Examples of Gradually Varied Flows		
10.6 Rapidly Varied Flow		
10.6.1 The Hydraulic Jump		
10.6.2 Sharp-Crested Weirs		
10.6.3 Broad-Crested Weirs		
10.6.4 Underflow Gates		
10.7 Chapter Summary and Study Guide		
Chapter 11 Compressible Flow (pp. 614-678 → 65 pages sub-total. 6 sections out of 8)		
11.3 Categories of Compressible Flow	623-628	6
11.4 Isentropic Flow of an Ideal Gas	631-646	16
11.4.2 Converging-Diverging Duct Flow		
11.4.3 Constant Area Duct Flow		
11.5 Non-isentropic Flow of an Ideal Gas	665-678	14
11.5.3 Normal Shock Waves		
11.6 Analogy between Compressible and Open-Channel Flows		
11.7 Two-Dimensional Compressible Flow		
11.8 Chapter Summary and Study Guide		
Chapter 12 Turbomachines (Whole Chapter; pp. 684-736 → 53 pages sub-total. 10 sections out of 10)		
12.1 Introduction	684-736	53
12.2 Basic Energy Considerations		
12.3 Basic Angular Momentum Considerations		
12.4 The Centrifugal Pump		
12.4.1 Theoretical Considerations		
12.4.2 Pump Performance Characteristics		
12.4.3 Net Positive Suction Head (NPSH)		
12.4 4 System Characteristics and Pump Selection		
12.5 Dimensionless Parameters and Similarity Laws		
12.5.1 Special Pump Scaling Laws		
12.5.2 Specific Speed		
12.5.3 Suction Specific Speed		
12.6 Axial-Flow and Mixed-Flow Pump		
12.7 Fans		
12.8 Turbines		
12.8.1 Impulse Turbines		
12.8.2 Reaction Turbines		
12.9 Compressible Flow Turbomachines		
12.9.1 Compressors		
12.9.2 Compressible Flow Turbines		

List 1A. (Continued)

Chapter/Section	Page Numbers	Number of Pages
Chapter 12 Turbomachines (Continued)		
12.10 Chapter Summary and Study Guide	↑	↑

Statistical Summary

Total Number of Pages Covered by Text (Excluding "Problems" Section)	621
Total Numbers of Sections Covered Under All Chapters	64 out of 102
Percentage of Pre-Calculus Sections $\%_{\text{Pre-Calculus}} = \left(\frac{\text{Number of Pre-Calculus Sections}}{\text{Total Number of Sections}}\right)(100\%) = \left(\frac{64}{102}\right)(100\%) = 62.7\%$	
Total Numbers of Chapters Covered	10 out of 12
Percentage of Chapters with Pre-Calculus Sections $\%_{\text{Pre-Calculus}} = \left(\frac{\text{Number of Chapters with Pre-Calculus Sections}}{\text{Total Number of Chapters}}\right)(100\%) = \left(\frac{10}{12}\right)(100\%) = 83.3\%$	
Total Number of Pages Covered by Pre-Calculus Portion	317
Percentage of Pre-Calculus Volume $\%_{\text{Pre-Calculus}} = \left(\frac{\text{Number of Pre-Calculus Pages}}{\text{Total Number of Pages}}\right)(100\%) = \left(\frac{317}{621}\right)(100\%) = 51.0\%$	